

If you want to quote this work:

Ostrowski, K. (2025). Tutoring research notebooks: Radio engineering - Course notes. Zenodo.

<https://doi.org/10.5281/zenodo.15341408>

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radiotechnika1.pdf

Radio engineering: Introduction

Radio technology tutoring

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Electromagnetic waves

Electromagnetic waves are disturbances in the electromagnetic field that propagate through space. They are described by Maxwell's equation and their fundamental characteristic is the transfer of energy without the need for a material medium.

Electric and magnetic waves

Electric waves are changes in the electric field, while magnetic waves are changes in the magnetic field. The two components are interrelated and oscillate perpendicular to each other and to the direction of wave propagation.

Maxwell's equations

Maxwell's equations describe the relationship between electric and magnetic fields. In differential form, they have the following form: $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ $\nabla \cdot \mathbf{B} = 0$ $\nabla \times \mathbf{E} = -\partial_t \mathbf{B}$ $\nabla \times \mathbf{B} = \mathbf{j} + \partial_t \mathbf{E}$.
 Understanding Maxwell's equations
 Maxwell's equations are the foundation of classical electrodynamics, describing how electric and magnetic fields affect each other and how they propagate. Formulated by James Clerk Maxwell, these four equations unify the theory of electromagnetism. Maxwell's equations can be written in the form of four differential equations, which we can understand more simply by analysing their physical meaning:
 * **First equation: Gauss's law for the electric field** $\nabla \cdot \mathbf{E} = \frac{\rho}{\epsilon_0}$ This equation states that the electric field \mathbf{E} is produced by electric charges (ρ) and its intensity is proportional to the charge in the area. The ratio ϵ_0 is the electrical permeability of the vacuum.
 * **Second equation: Gaussian law for the magnetic field** $\nabla \cdot \mathbf{B} = 0$.

This equation states that the magnetic field \mathbf{B} has no sources or poles, meaning that it always forms loops - there are no 'magnetic charges'.

- **Third equation: Faraday's law of electromagnetic induction** $\nabla \times \mathbf{E} = - \frac{\partial \mathbf{B}}{\partial t}$. This equation says that a change over time in the magnetic field \mathbf{B} produces an electric field \mathbf{E} . This is the principle of induction currents, such as in generators.
- **Fourth equation: Ampère's law with Maxwell's correction** $\nabla \times \mathbf{B} = \mathbf{J} + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t}$. This equation says that the electric current \mathbf{J} and the changing electric field \mathbf{E} create a magnetic field \mathbf{B} . The index \mathbf{J} is the magnetic permeability of the vacuum.

These equations show how changing electric and magnetic fields affect each other and how they create electromagnetic waves. In practice, this means that a varying electric field produces a magnetic field and a varying magnetic field produces an electric field, which leads to the propagation of an electromagnetic wave, such as light.

Understanding Maxwell's equations is that these four equations show how electromagnetic energy is generated and transmitted through space. We can think of them as 'coupled' equations that complement and interact with each other.

The carrier of the electromagnetic wave: the photon

The photon is an elementary particle that is the carrier of an electromagnetic wave. According to quantum theory, the photon has no rest mass, but has energy and momentum. The photon's energy is related to the frequency of the electromagnetic wave and its momentum depends on the wavelength. This relationship is described by the equation:

$E = h \nu$ where E is the energy of the photon, h is Planck's constant, and ν is the frequency of the electromagnetic wave.

Furthermore, the momentum of the photon is expressed by the equation:

$p = \frac{h}{\lambda}$ where p is the photon's momentum and λ is the wavelength.

The photon travels at the speed of light in a vacuum, which is $c = 3 \times 10^8 \text{ m/s}$. In the context of an electromagnetic wave, a photon is represented as a quantum of energy that carries both energy and momentum depending on its frequency and polarisation.

The formula for the energy of a photon can also be written in the form:

$E = hc / \lambda$.

It is worth noting that the photon is described by both the properties of the particle (energy, momentum) and the properties of the wave (frequency, wavelength), which is the basis of corpuscular-wave duality.

The dual nature of light and interference

Light exhibits both wave and corpuscular characteristics. Interference, the overlapping of waves, leads to the amplification or extinction of the signal depending on the phase.

Dual nature of light: explanation

The dual nature of light refers to the fact that light exhibits the properties of both a wave and a particle, depending on how it is studied. This concept has revolutionised our understanding of the nature of light and has been central to the development of quantum physics.

The discovery of duality

The concept of the duality of light developed gradually over the centuries, and one of the most important moments was the discovery that light could manifest both wave and particle characteristics. In the 19th century, research into the nature of light by a number of scientists led to the discovery that light has wave-like characteristics.

- **Thomas Young (1801)**: He performed a double slit experiment that showed that light interferes, which is characteristic of waves. This experiment was proof of the wave nature of light.
- **James Clerk Maxwell (1864)**: Developed theories that described light as an electromagnetic wave. From his Maxwell's equations, it was clear that light was an electromagnetic wave with a specific frequency and wavelength.
- **Albert Einstein (1905)**: In his study of the photoelectric effect, Einstein proposed that light could also be treated as a stream of particles called photons. His work showed that light has the properties of a particle, because only energy of a certain value (quantum) can release electrons from the surface of a metal. He was awarded the Nobel Prize in Physics in 1921 for this discovery.

Why is the duality of light important?

The duality of light is central to modern physics because it shows that in the microscale world (at the particle level), the classical concepts of 'particle' and 'wave' are not sufficient to describe phenomena. Light, as well as other elementary particles (e.g. the electron), exhibit both aspects depending on the type of experiment. For this reason, quantum physics has become necessary to fully understand these phenomena.

In short, the discovery of the duality of light was a watershed moment that pointed out the limitations of classical theories and gave rise to a new, more comprehensive theory, which is quantum physics.

Interference in a double slit experiment

The double slit experiment demonstrates how light can form characteristic interference striations. These striations are the result of light waves from two slits overlapping and strengthening or weakening each other depending on the difference in the paths they take.

The mathematics of interference

Formula describing the positions of interference bands:

$$y_m = \frac{m \lambda L}{d}$$

where:

- y_m - position of the m-th stripe on the screen,
- λ - wavelength,
- L - distance from slits to screen,
- d - distance between slits,
- m - bar number (where $m = 0$ is the central bar, $m = 1$ for the first bars, etc.).

Graph of interference with striations

To get clear striations, we can use a sinusoidal function that mimics the effect of interference, generating a sharp pattern of striations. We will modify the graph to show these striations in a more pronounced way.



Applications

The diagram shows the characteristic arrangement of interference striations, with bright striations

occurring where the path difference is a multiple of the wavelength, and dark striations occurring where the path difference is half a wavelength (or multiples thereof).

Electromagnetic spectrum

The electromagnetic spectrum includes radio waves, microwaves, infrared, visible light, ultraviolet, X-rays and gamma rays. Low-frequency waves have longer ranges and better penetration of obstacles, while high-frequency waves have a greater ability to transmit information.

Ranges of electromagnetic radiation

Electromagnetic radiation covers a wide range of wavelengths and frequencies, which determines its various properties and applications. A brief description of the different ranges is given below:

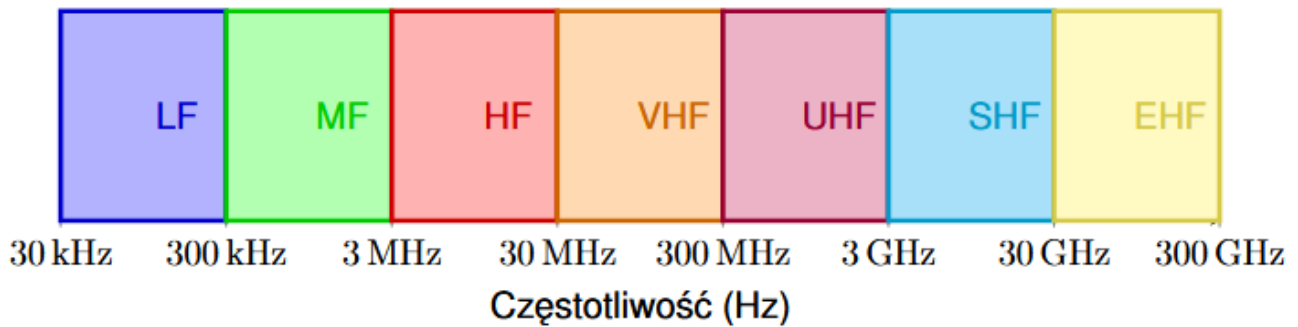
- **Radio waves:** Wavelengths from a few millimetres to thousands of metres. Used in radio transmission, television, satellite communications and radar.
- **Microwaves**Wavelengths in the order of centimetres. Used in communication systems, radar technology and microwave ovens.
- **Infrared (IR)**Wavelengths from approximately 700 nm to 1 mm. Responsible for heat emission; used in thermal imaging, remote control and diagnostics.
- **Visible light:** Wavelength range from approximately 380 nm (violet) to 750 nm (red). This is the band that is visible to the human eye and responsible for colour perception.
- **Ultraviolet (UV):** Wavelengths from approximately 10 nm to 380 nm. Used in medicine, disinfection and scientific research, although overexposure can be harmful.
- **X-rays (X-ray).**: Wavelengths from about 0.01 nm to 10 nm. Used in medical diagnostics (e.g. X-rays) and structural analysis of materials.
- **Gamma radiation**Wavelengths of less than 0.01 nm. It is characterised by very high energy, which makes it used in cancer therapy, astrophysical research and materials testing, but it is also a potential health hazard.

Radio spectrum

Radio radiation is the lowest range of electromagnetic radiation with the longest wavelengths and lowest frequencies. Depending on the wavelength, radio waves are divided into different ranges, which have different applications in technology and science. Radio waves are used in communications, navigation, radar, telecommunications and many other fields.

Chart of radio bands

The chart below shows the different radio bands on the frequency axis (in Hz) on a logarithmic scale. Each band is shown as a rectangle of a different colour, with the band name in the centre.



Radio wave bands

Radio waves are divided into several main bands, depending on wavelength and frequency:

- **Long wave (LF, Long Wave):** Wavelengths range from 1 km to 100 km and frequencies from 30 kHz to 300 kHz. These waves are used in navigation and communication systems, such as amateur radio. They are characterised by their ability to travel long distances, especially with reflections from the ionosphere.
- **Medium wave (MF, Medium Wave):** Wavelengths range from 100 m to 1 km and frequencies range from 300 kHz to 3 MHz. They are mainly used in AM (amplitude modulation) transmission in radio. MF waves penetrate the atmosphere well, but their range is limited.
- **Short wave (HF, High Frequency):** Wavelengths range from 10 m to 100 m and frequencies are in the 3 MHz to 30 MHz range. These waves have the ability to bounce off the ionosphere, allowing long distance communication. They are used in international communications, amateur radio and also in meteorological systems.
- **Very short wave (VHF, Very High Frequency):** Wavelengths range from 1 m to 10 m and frequencies range from 30 MHz to 300 MHz. These waves are used in radio communications (e.g. FM radio), television (e.g. analogue TV broadcasting), aeronautical, maritime communications systems and mobile radio communications.
- **Ultra high frequency (UHF):** Wavelengths range from 10 cm to 1 m and frequencies range from 300 MHz to 3 GHz. UHF waves are used in mobile phone communications, digital television, radar and GPS systems. They have good building penetration, but their range is less than for VHF waves.
- **Super High Frequency (SHF):** Wavelengths range from 1 cm to 10 cm and frequencies range from 3 GHz to 30 GHz. SHF waves are used in radar systems, satellite communications, Wi-Fi wireless communications and 5G systems.
- **Extremely High Frequency (EHF) waves:** Wavelengths range from 1 mm to 1 cm and frequencies range from 30 GHz to 300 GHz. EHF waves are used in satellite communication systems, space radio communications and 5G technology in very high frequency bands.

Technological applications of radio waves

Radio waves have a wide range of applications in various technological fields:

- **Radio and television:** Long-wave (LF), medium-frequency (MF) and very short-wave (VHF) are used in AM, FM radio transmission and in analogue and digital television.
- **Telecommunications and mobile communications:** UHF and SHF waves are the basis for

mobile phone systems (e.g. LTE, 5G), as well as for Wi-Fi and Bluetooth systems. With the right frequencies, fast and reliable data transmission is possible.

- **Navigation and GPS systems:** Radio waves, mainly in the UHF and SHF range, are used in satellite navigation systems, such as GPS (Global Positioning System), which make it possible to determine the exact position on Earth.
- **Radar and military communications:** SHF and EHF waves are used in radar systems used in aviation, navy and military applications for object detection, among others. These waves are also used for communication in space.
- **Wi-Fi and wireless communications:** SHF waves, particularly in the 2.4 GHz and 5 GHz bands, are used in Wi-Fi technology, which enables wireless data transmission in local area networks.
- **Amateur radio communications:** Short-wave (HF) and ultra-short-wave (VHF) are used by radio amateurs around the world for communications, both for local and international transmissions.
- **Microwave ovens:** Microwaves (2.45 GHz) are used in microwave ovens to heat food, thanks to their specific interaction with water and fat molecules.

Properties of radio waves

Radio waves have different physical properties, depending on their wavelength and frequency:

- **Scattering and reflection:** Radio waves can reflect off the Earth's surface, atmosphere or ionosphere, allowing long-range transmission of signals. Low-frequency waves (e.g. long-wave) can travel a very long distance by reflecting off the ionosphere.
- **Penetration:** UHF and SHF waves have a better ability to penetrate obstacles, such as building walls, compared to lower frequency waves (e.g. longwave).
- **Range:** Low-frequency waves (LF, MF) have a long range, up to several thousand kilometres, as they are able to reflect off the ionosphere. Higher frequency waves (VHF, UHF) have a shorter range but better signal quality.
- **Atmospheric absorption:** In the case of very high frequency waves (SHF, EHF), their range is limited by absorption by the atmosphere, rain and other meteorological factors.

Summary

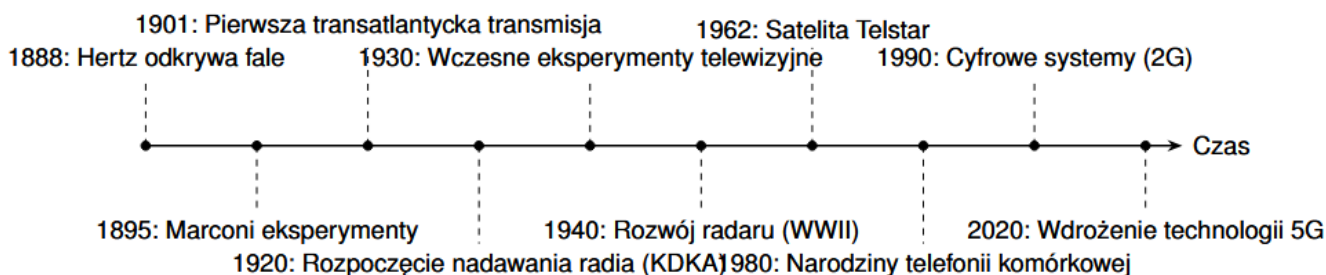
Radio radiation is a very wide range of frequencies that are used in almost every field of modern technology. Radio waves are used in telecommunications, radar, navigation, communication systems, as well as in everyday devices such as Wi-Fi and microwave ovens. Each radio wave range has its own specific properties and applications, which are used for different purposes depending on their frequency and wavelength.

History of radio technology

Radio technology has a rich history, dating back to the late 19th century. From the first experiments with electromagnetic waves to modern wireless communication technologies, the development of radio technology exemplifies the dynamic technological advances that have changed the way the

world communicates.

Timeline of revolutionary discoveries in radio technology



Origins of radio technology: Guglielmo Marconi (1895-1901)

The first research into electromagnetic waves was carried out by scientists such as Heinrich Hertz, who proved the existence of radio waves in 1888. However, it is Guglielmo Marconi who is recognised as the creator of the first radio technology. In 1895, Marconi built the first radio transmission system, using equipment to transmit and receive signals via radio waves.

In 1901, Marconi carried out the first transatlantic radio transmission, broadcasting a signal from England to Canada. His experiments paved the way for the development of wireless and long-distance communication.

Arc transmitters and the development of communications (1900-1920)

In the early days of radio technology, transmitting devices used so-called arc transmitters. These transmitters were based on electric arcs that produced radio waves. Although they were effective, they had a number of disadvantages, such as instability and difficulty in obtaining the required frequencies. Nevertheless, these devices were a key element in the development of early radio communications.

In the 1920s, with the help of better technology and more advanced transmitters, such as tube transmitters and more precise resonant circuits, more stable transmission systems began to be developed. This enabled the development of the first regular radio transmissions, as well as the birth of public radio.

Development of radio and television (1920-1950)

In the 1920s, the first public radio broadcasts began. In 1920, the first radio station in the United States, KDKA in Pittsburgh, began broadcasting regular music and news programmes. At the same time in Europe, radio stations began to operate in the UK, France and other countries.

The 1930s and 1940s were a time of intense growth for radio. Radio became a mass communication medium, used not only for entertainment but also for news, especially during the Second World War. At the same time, the first television experiments began to appear in the 1930s, which later became widely available in the 1950s.

Radar and satellite technology (1950-1980)

After the Second World War, radio technology began to be widely used in the military, especially in radar systems, which enabled the detection of objects in the airspace. Radar technology used radio waves at different frequencies, which allowed precise determination of distances and directions.

In the 1960s, on the other hand, radio technology began to be used in satellite communication systems. The first communication satellites, such as Telstar (1962), revolutionised international communication, allowing television, telephone and radio signals to be transmitted over long distances.

Mobile telecommunications and the development of radio bands (1980-2000)

From the 1980s onwards, mobile telecommunications technologies began to develop, which used radio waves to transmit voice and data over long distances. In the first half of the 1980s, the first mobile phone systems (1G) were introduced, which used analogue radio waves to transmit telephone calls. In the 1990s came digital systems (2G), which also allowed data transmission, including short message service (SMS).

The late 20th century also saw the development of Wi-Fi technology, which uses the radio bands in the UHF and SHF range to enable wireless access to the Internet.

Modern radio technologies (2000 - present)

In the 21st century, radio technology has reached unimaginable levels of sophistication. The main developments have been the development of 3G, 4G and 5G networks, which have allowed high-speed internet access, high-definition video transmission and the development of IoT (Internet of Things) technology.

5G networks, which came into use in the early 2020s, offer extremely fast internet connections, low latency and the ability to support millions of devices in a single area. The technology uses radio bands over a very wide range of frequencies, including millimetre bands (30 GHz - 100 GHz), which allow very high transmission speeds.

Developments in satellite communications technology, including systems such as Starlink, are making it possible to provide global access to the internet, and radar technology continues to be used in areas ranging from airspace monitoring to autonomous cars.

Summary

The history of radio technology is one of continuous development that has impacted our everyday lives, changing the way we communicate, work and use technology. From Marconi's first experiments to today's 5G and satellite systems, radio waves have become the foundation of modern telecommunications and wireless communications. Thanks to these technologies, the world has become more connected and communication capabilities have developed in ways that seemed impossible just a century ago.

Reactance and impedance

Reactance is part of impedance and depends on frequency: $X_L = \omega L$, $X_C = \frac{1}{\omega C}$ While impedance is the sum of resistance and reactance: $Z = R + jX$

Explanation

Reactance and impedance are basic concepts in electrical circuits, especially in alternating current (AC) circuits. They are used to describe the resistance that circuit components place on the flow of alternating current. Although they are related, they are different.

Reactance

Reactance (X) is the resistance that results from the presence of inductive (L) and capacitive (C) elements in a circuit. Reactance is not a resistance in the classical sense (as in the case of a resistor) because it does not lead to a complete dissipation of energy in the form of heat, but only causes a retardation or acceleration of current flow relative to voltage.

Inductive reactance (denoted X_L) and capacitive reactance (denoted X_C) are described by the formulae:

$$X_L = \omega L \quad X_C = \frac{1}{\omega C}$$

where:

- $\omega = 2\pi f$ is the pulsation (angular frequency),
- f is the frequency of the alternating current,
- L is the inductance of the coil,
- C is the capacitance of the capacitor.

The inductive reactance increases as frequency increases, while the capacitive reactance decreases as frequency increases.

Impedance

Impedance (Z) is a complex quantity that describes the total resistance in AC circuits, taking into account both resistive resistance (R) and reactance (X).

Impedance is expressed by the formula:

$$Z = R + jX$$

where:

- R is the resistive resistance (constant over time),
- X is the reactance (frequency dependent),
- j is the imaginary unit ($j^2 = -1$).

Impedance is a complex quantity, which means that, in addition to the impedance modulus (which can be thought of as a 'resistance quantity'), there is also its argument, which describes the phase shift between voltage and current. The impedance modulus ($|Z|$) is defined as:

$$|Z| = \sqrt{R^2 + X^2}$$

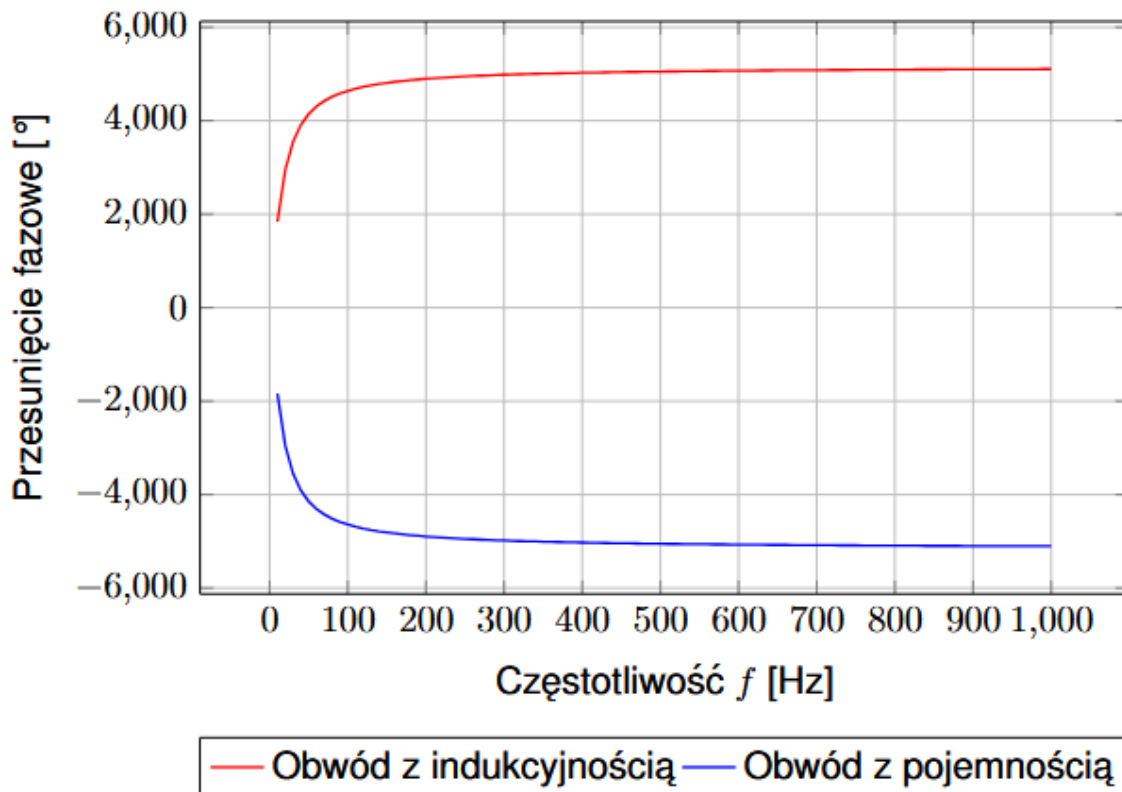
In contrast, the impedance argument ($\arg(Z)$) gives the phase shift between voltage and current and is equal to:

$\arg(Z) = \arctan\left(\frac{X}{R}\right)$.
 Understanding how reactance and impedance work
 To better understand the operation of reactance and impedance, it is useful to know how these quantities affect the behaviour of current in AC circuits. * Inductive reactance (X_L) causes the current to delay relative to the voltage. In circuits with inductive components, the alternating current encounters 'resistance' in the form of a changing magnetic field, leading to a delay in current flow.

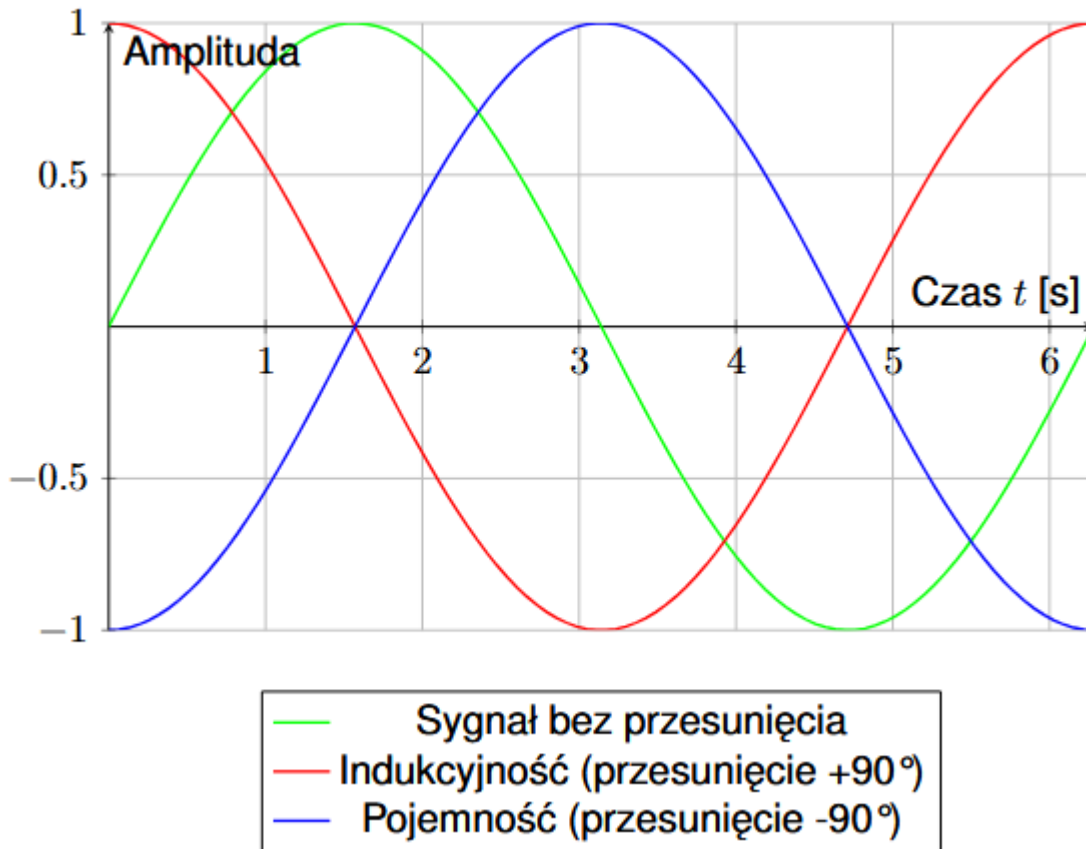
- Capacitive reactance (X_C) works in the opposite way. In circuits with capacitive elements, the current 'precedes' the voltage because the capacitor stores energy in the form of an electric field and then releases it, leading to an acceleration of current flow.
- Impedance, which is a combination of resistance and reactance, allows a more complete description of the effect of different circuit elements on current flow. Understanding impedance is crucial in the design of AC circuits, especially in areas such as telecommunications, electronics and power engineering.

Phase shifts in circuits with capacitive and inductive reactance

Przesunięcie fazowe w funkcji częstotliwości dla obwodów L i C



Przesunięcie fazowe sygnałów w domenie czasu



Inductive reactance

The inductive reactance (X_L) causes the current to lag relative to the voltage. This relationship is described by the formula:

$X_L = \omega L$ where $\omega = 2\pi f$ is the pulsation, L is the inductance of the coil and f is the frequency of the alternating current. The phase shift for the inductive reactance is $\frac{\pi}{2}$ (90°) because the current is delayed by a quarter of a period relative to the voltage.

Capacitive reactance

Capacitive reactance (X_C) causes the current to be ahead of the voltage. This relationship is described by the formula:

$X_C = \frac{1}{\omega C}$ where C is the capacitance of the capacitor. The phase shift in capacitive circuits is $-\frac{\pi}{2}$ (-90°), because the current is ahead of the voltage by a quarter of a period.

Phase shift as a function of frequency for circuits with inductance and capacitance.

Sinusoidal signals with phase shift due to inductance, capacitance and no shift in the time domain.

Explanation of phase shifts

In AC circuits, passive elements such as inductors and capacitors cause a phase shift between voltage and current. This phenomenon is due to the nature of these elements:

- Inductive reactance: In a circuit with an inductor, the current is delayed with respect to the voltage by $\frac{\pi}{2}$ (90°). This is due to the fact that the coil resists the current flow, creating a magnetic field that cannot immediately respond to changes in voltage.
- * Capacitive reactance: In a circuit with a capacitor, the current precedes the voltage by $\frac{\pi}{2}$ (-90°). The capacitor stores electrical charge, which causes the voltage on the capacitor to change with a delay relative to the current.

The phase shift diagram shows that for an inductive reactance the phase shift is $+90^\circ$, while for a capacitive reactance it is -90° , which corresponds to a delay or advance of the current relative to the voltage.

Dependence of reactance and impedance on frequency

Reactance and impedance are key quantities in AC circuits. Their value depends on the frequency of the alternating current, which is important when analysing circuits with passive elements such as inductors (inductance) and capacitors (capacitance).

Inductive reactance

Inductive reactance (X_L) is the resistance provided by an inductor in an AC circuit. An increase in the frequency of the alternating current results in an increase in inductive reactance, as the coil puts more and more resistance to the current flow as the rate of change of the magnetic field increases.

Inductive reactance is described by the formula:

$$X_L = \omega L = 2\pi f L$$

where:

- $\omega = 2\pi f$ is the pulsation,
- f is the frequency of the alternating current,
- L is the inductance of the coil.

This formula shows that the inductive reactance increases linearly with increasing frequency f . For higher frequencies, the coil becomes increasingly 'resistant' to current flow.

Capacitive reactance

Capacitive reactance (X_C) is the resistance put up by a capacitor in an AC circuit. An increase in the frequency of the AC current results in a decrease in capacitive reactance, as the capacitor charges and discharges faster in higher frequency circuits.

Capacitive reactance is described by the formula:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

where:

- C is the capacitance of the capacitor,
- f is the frequency of the alternating current.

From this formula it follows that the capacitive reactance decreases inversely proportional to the frequency f . For higher frequencies, the capacitor becomes more 'permeable' to current.

Impedance

Impedance (Z) is a complex quantity that describes the total resistance in an AC circuit. It includes both resistive resistance (R) and reactance (X). For circuits consisting of an inductor and a capacitor, the impedance depends on the frequency of the alternating current, as both inductive and capacitive reactance depend on frequency.

The impedance in circuits with both inductive and capacitive reactance is expressed as:

$$Z = R + jX$$

where j is the imaginary unit ($j^2 = -1$), R is the resistive resistance and X is the reactance, which can be inductive (X_L) or capacitive (X_C).

For circuits consisting of an inductor and a capacitor, the impedance varies with frequency:

- For an inductive circuit, the impedance increases linearly with frequency. - For a capacitive circuit, the impedance decreases inversely proportional to frequency.

The impedance modulus (which is 'resistance' in the physical sense) is expressed by the formula:

$$|Z| = \sqrt{R^2 + X^2}$$

For an LC circuit (with an inductor and a capacitor), where $X_L = X_C$, the impedance is minimum at the point of resonance, i.e. when the inductive reactance is equal to the capacitive reactance ($X_L = X_C$).

Summary of the relationship

To summarise:

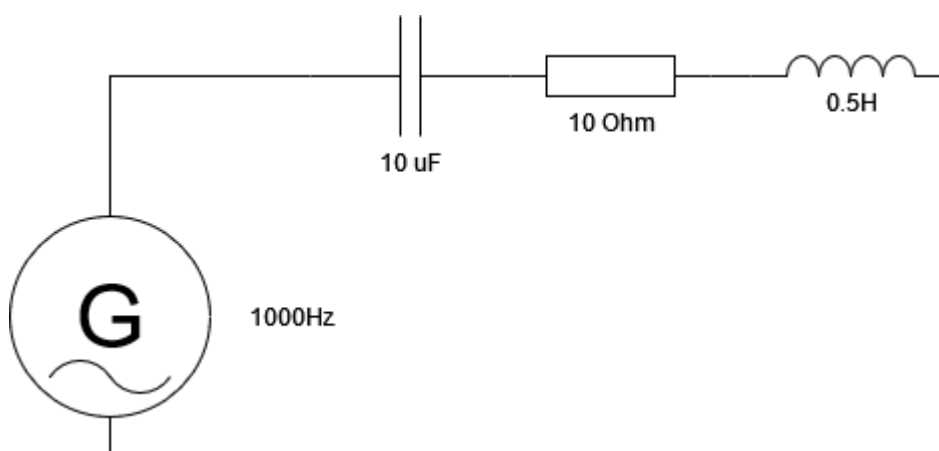
- Inductive reactance increases linearly with frequency ($X_L \propto f$).
- Capacitive reactance decreases inversely proportional to frequency ($X_C \propto \frac{1}{f}$).
- Impedance depends on frequency through the components of reactance and resistance. For LC circuits, impedance is minimum at resonance when $X_L = X_C$.

RLC circuit in series - impedance and reactance

Consider a series RLC circuit consisting of a resistor R , an inductor L and a capacitor C . Such a circuit is a basic example of an AC circuit that has an impedance-frequency relationship.

Schematic of a series RLC circuit

The schematic of a series RLC circuit is as follows:



Calculation of impedance and reactance

The impedance Z of a series RLC circuit is a complex quantity and is described by the formula:

$$Z = R + jX$$

where:

- R is the resistive resistance,
- X is the reactance, which is the sum of the inductive reactance X_L and the capacitive reactance X_C , i.e. $X = X_L - X_C$.

The inductive reactance X_L is given by the formula:

$$X_L = \omega L = 2\pi f L$$

The capacitive reactance X_C is given by the formula:

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi f C}$$

where:

- $\omega = 2\pi f$ is the pulsation,
- f is the frequency of the alternating current,
- L is the inductance of the coil,
- C is the capacitance of the capacitor.

The total impedance of a series RLC circuit is therefore the sum of the resistance R and the difference of the inductive and capacitive reactances:

$$Z = R + j(X_L - X_C)$$

Example of a calculation for a frequency $f = 1000 \text{ Hz}$.

Suppose we have the following parameters:

$$R = 10 \text{ } \Omega, \quad L = 0.5 \text{ mH}, \quad C = 10 \text{ nF}, \quad f = 1000 \text{ Hz}$$

Let's first calculate the reactances:

$$X_L = \omega L = 2\pi f L = 2\pi \cdot 1000 \cdot 0.5 \cdot 10^{-3} = 3141.6 \text{ } \Omega$$

$$X_C = \frac{1}{\omega C} = \frac{1}{2\pi \cdot 1000 \cdot 10 \cdot 10^{-9}} = 15.92 \text{ n}\Omega$$

The total reactance X is:

$$X = X_L - X_C = 3141.6 - 15.92 = 3125.68 \text{ } \Omega$$

The impedance of Z is:

$$Z = R + jX = 10 + j \cdot 3125.68 \text{ } \Omega$$

The impedance modulus is:

$$|Z| = \sqrt{R^2 + X^2} = \sqrt{10^2 + 3125.68^2} \approx 3125.7 \text{ n}\Omega$$

Summary

The impedance of a series RLC circuit as a function of frequency depends on the components of the circuit, including inductance, capacitance and resistance. In this example, we have calculated the impedance for a specific frequency $f = 1000 \text{ Hz}$, obtaining a value of $|Z| \approx 3125.7 \text{ }\mu\Omega$, indicating a high dominance of inductive reactance in this circuit. The value of the impedance can vary with frequency, since the reactances X_L and X_C are functions of frequency.

Polarisation of an electromagnetic wave

Electromagnetic waves consist of oscillating vectors of the electric field \mathbf{E} and the magnetic field \mathbf{B} (or \mathbf{M} in some cases), which are closely related. Polarisation refers to the orientation of the vector \mathbf{E} in space.

Electric and magnetic field vectors

An electromagnetic wave can be described by wave equations, which express the relationship between the electric and magnetic fields. In three-dimensional space, the electric and magnetic fields are perpendicular to each other and perpendicular to the direction of propagation of the wave.

$$\mathbf{E} = E_0 \cos(kz - \omega t) \hat{\mathbf{e}}_E \quad \mathbf{B} = B_0 \cos(kz - \omega t) \hat{\mathbf{e}}_B$$

where: - E_0 and B_0 are the amplitudes of the electric and magnetic fields, - k is the wave vector, - ω is the angular frequency, - $\hat{\mathbf{e}}_E$ and $\hat{\mathbf{e}}_B$ are the unit vectors of the electric and magnetic field directions.

An electromagnetic wave propagates in the direction $\hat{\mathbf{z}}$ and is described by sinusoidal functions.

Linear, circular and elliptical polarisation

Electromagnetic waves can be polarised in different ways:

Linear polarisation:

In the case of linear polarisation, the vector \mathbf{E} oscillates in one plane, e.g. along the x -axis. This polarisation occurs when the vector \mathbf{E} remains constant in a given direction.

Mathematically, for linear polarisation: $\mathbf{E}(t) = E_0 \cos(\omega t) \hat{\mathbf{e}}_E$.

Circular polarisation:

In the case of circular polarisation, the vector \mathbf{E} performs a circular motion in a plane perpendicular to the direction of wave propagation. The \mathbf{E} vector changes its orientation over time, but its amplitude remains constant.

Mathematically, for circular polarisation: $\mathbf{E}(t) = E_0 \cos(\omega t) \mathbf{e}_x + E_0 \sin(\omega t) \mathbf{e}_y$.

Elliptical polarisation:

Elliptical polarisation is a generalisation of circular polarisation, in which the vector \mathbf{E} describes an ellipse in space. It is a combination of two sinusoidal components with different amplitudes and phase shifts.

Mathematically, for an elliptical polarisation: $\mathbf{E}(t) = E_0 \cos(\omega t) \mathbf{e}_x + E_0 \sin(\omega t + \phi) \mathbf{e}_y$.

where $\phi_1 \neq \phi_2$ leads to an elliptical shape.

H, V, and right- and left-handed circular polarisation

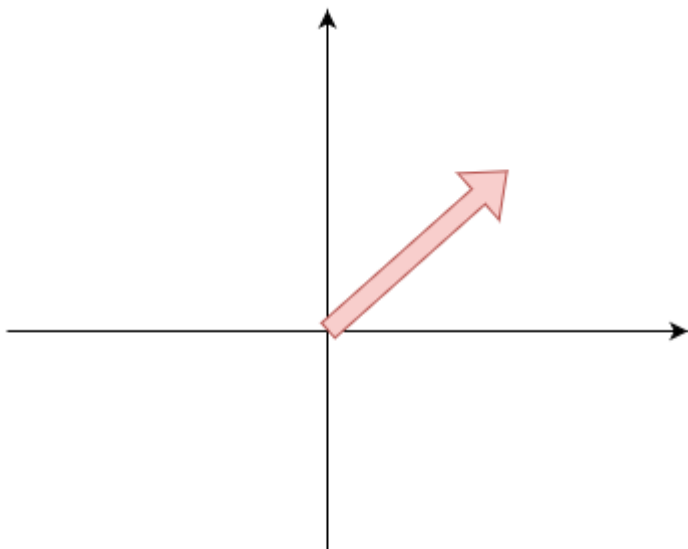
- Horizontal polarisation (H): The vector \mathbf{E} oscillates in the horizontal plane.
- Vertical polarisation (V): The \mathbf{E} vector oscillates in the vertical plane.
- Right- and left-hand circular polarisation: Determines which way the vector \mathbf{E} rotates in time. Right-handed (right-handed) or left-handed (left-handed) polarisation depends on the direction of rotation of the vector \mathbf{E} . Right-handed polarisation means clockwise rotation and left-handed means counterclockwise rotation.

Polarisation diagrams

In this section we show the different types of polarisation of electromagnetic waves.

Linear polarisation

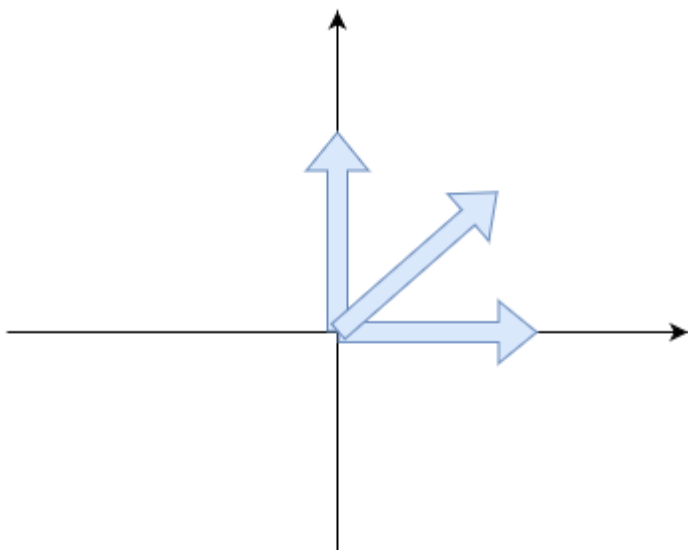
In the case of linear polarisation, the vector \mathbf{E} oscillates in one fixed plane. The diagram below shows the motion of the vector \mathbf{E} along the x axis, where the vector does not change its orientation in time.



The electric field vector \mathbf{E} in the case of linear polarisation.

Circular polarisation

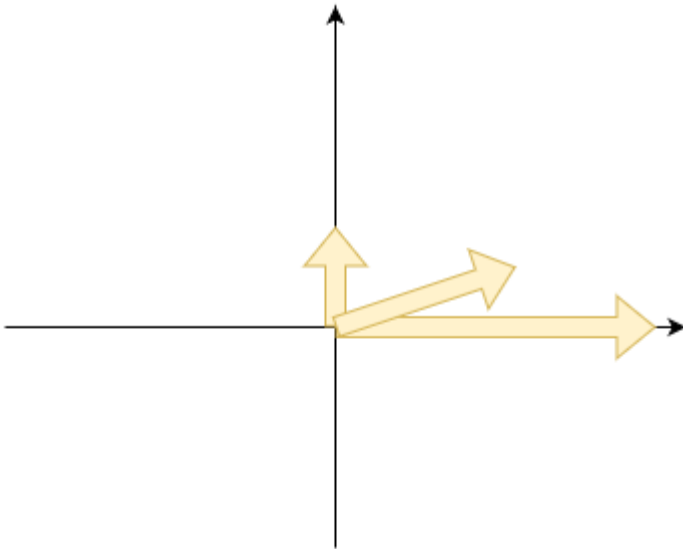
In the case of circular polarisation, the vector \mathbf{E} moves along a circle in a plane perpendicular to the direction of wave propagation. The diagram below shows two time instants at which the vector \mathbf{E} is at two different positions in the rotating circle.



The electric field vector \mathbf{E} in the case of circular polarisation.

Elliptical polarisation

Elliptical polarisation is the more general case in which the vector \mathbf{E} moves along an ellipse. The components E_x and E_y have different amplitudes and phases, resulting in an elliptical trajectory for the motion of the vector \mathbf{E} . The diagram below shows the motion of the vector \mathbf{E} along the ellipse.



Electric field vector \mathbf{E} in the case of elliptical polarisation.

These diagrams illustrate the basic types of polarisation, showing the differences in the motion of the \mathbf{E} vector in space. Linear polarisation is the simplest case, where the \mathbf{E} vector oscillates in a single plane, while circular and elliptical polarisation show more complex dynamics.

The use of different types of polarisation

The polarisation of electromagnetic waves plays a key role in many areas of technology, communication and science. The type of polarisation affects how electromagnetic waves interact with materials, devices and how they are transmitted through space. In this section, we discuss when and why different types of polarisation are used, such as linear, circular and elliptical polarisation.

Linear polarisation

Linear polarisation is the simplest and most commonly used type of polarisation, in which the electric field vector oscillates in a single plane. It is a type of polarisation in which an electromagnetic wave has a constant polarisation direction at a given point in space.

Application

- **Radio and television communications:** In telecommunications, particularly in radio, satellite and television transmission systems, linear polarisation is commonly used. Waves emitted from transmitters have a specific polarisation, which is maintained throughout the propagation path. In linear polarisation it is easier to control transmission, eliminate reflections and reduce interference.
- **Antennas:** Many antennas, such as dipole antennas and Yagi antennas, operate in linear polarisation, where the electric field vector produces signals in one plane. Using linear polarisation makes it easier to obtain strong and unambiguous signals.

When using linear polarisation

Linear polarisation is most commonly used when there is no need to change the orientation of the electric field vector over time. It works best in simple communication applications where the transmission channel (e.g. air) does not cause significant changes in polarisation characteristics.

Circular polarisation

Circular polarisation is a case of polarisation in which the electric field vector rotates around the axis of wave propagation, forming a circular trajectory. Circular polarisation can be right or left, depending on the direction of rotation.

Application of

- **Satellite links:** In satellite communications, especially when communicating with satellites, circular polarisation (right- or left-handed) is used to eliminate interference. Satellites can receive signals from two different sources at different circular polarisations (right and left), allowing the same frequency to be used effectively for different transmissions.
- **Radar:** In radar technology, circular polarisation is used to better distinguish objects in different atmospheric conditions and to increase the efficiency of detecting objects in different configurations.
- **Photocells and sensors:** In some photonics applications, such as photocells, circular polarisation can be used to control the intensity of light and to precisely monitor the interaction of light with materials.

When we use circular polarisation

Circular polarisation is preferred when there is a need for increased resistance to reflections, and when the same frequency is used for different transmission channels. Circular polarisation is also ideal for eliminating interference caused by changing weather conditions and surface reflections.

Elliptical polarisation

Elliptical polarisation is the most general case of polarisation, in which the electric field vector moves along an ellipse. In this case, the components E_x and E_y have different amplitudes and phases, leading to a more complex trajectory.

Application of

- **Electromagnetic field theories:** Elliptical polarisation is widely used in the analysis and modelling of physical phenomena such as wave scattering by materials and various substances.
- **Lasers:** In some laser applications, elliptical polarisation is used to precisely control the scattering and reflection of light.
- **Polarimetric camera:** In polarimetric cameras, elliptical polarisation allows more accurate

mapping of changes in matter and on the surface of objects, which is useful in medicine, life sciences and geophysical research.

When we use elliptical polarisation

Elliptical polarisation is used when more complex interactions with matter are required, including, for example, cases of materials analysis, measurements using laser detection systems, or optical technologies.

Summary

Depending on the specific application, different types of electromagnetic wave polarisation can be used to improve performance, eliminate interference, or accurately model wave-environment interactions. Linear polarisation is most commonly used in telecommunications and antennas, circular polarisation in satellite communications and radar, and elliptical polarisation in scientific applications and photonics. Understanding these types of polarisation allows the design of more efficient communication and technological systems.