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Radio engineering: Analysis of analogue modulation, digital modulation, transimpedance, BalUn and UnUn circuits

Radio engineering tutoring

Modulations of radio signals

Modulation of a signal involves changing one of its characteristics (amplitude, frequency or phase) according to the information contained in the modulating signal. In this section we will discuss amplitude modulations.

Amplitude modulations

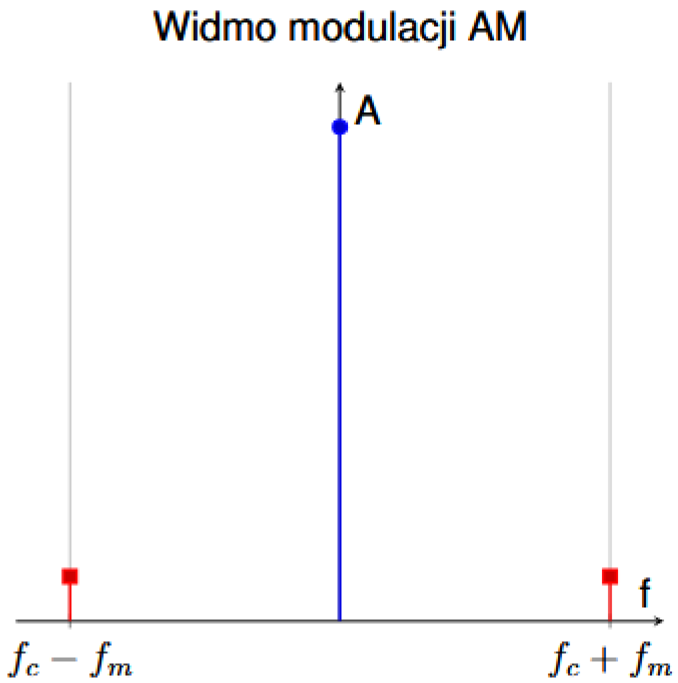
In amplitude modulations, the carrier changes its amplitude according to the modulating signal.

Normal AM modulation

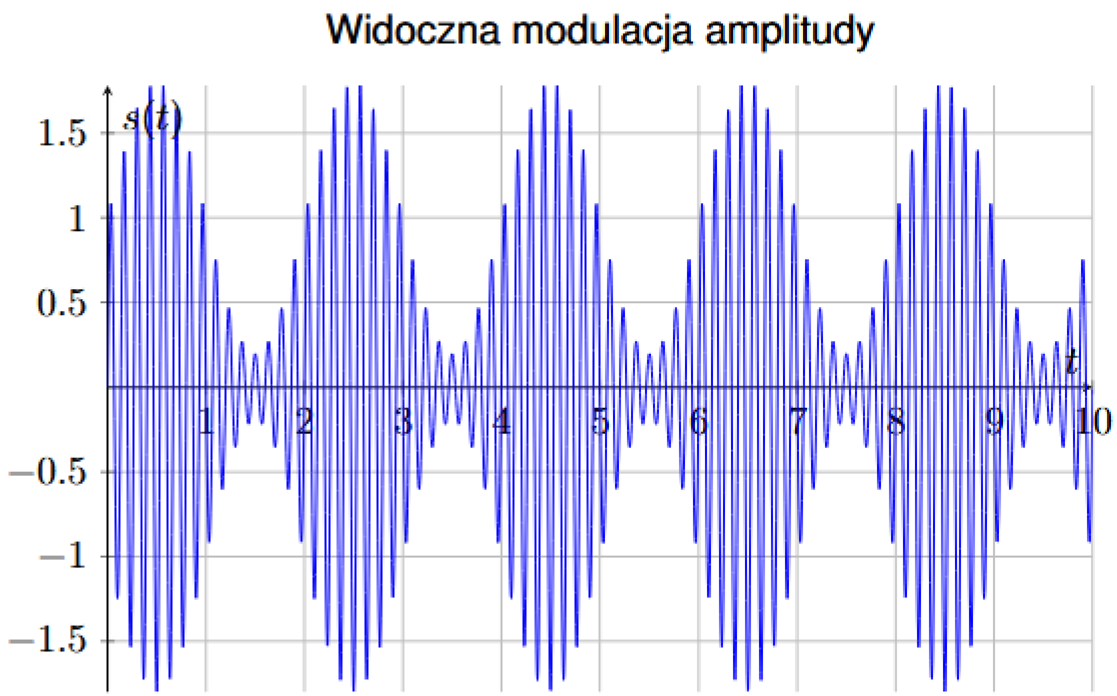
In classical amplitude modulation (AM), the modulated signal has the form: $$s(t) = \big(A + m(t) \big) \cos(2 \pi f_c t),$$ where:

- A - carrier amplitude,
- $m(t)$ - modulating signal,
- f_c - carrier frequency.

The spectrum of an AM signal consists of a carrier wave and two sidebands spaced by the frequency of the modulating signal.



AM signal spectrum



AM signal graph

DSB-AM modulation

Dual sideband modulation (DSB-AM) is one form of amplitude modulation in which both sidebands are transmitted without a carrier wave. The expression for the signal modulated in this form is described by the formula:

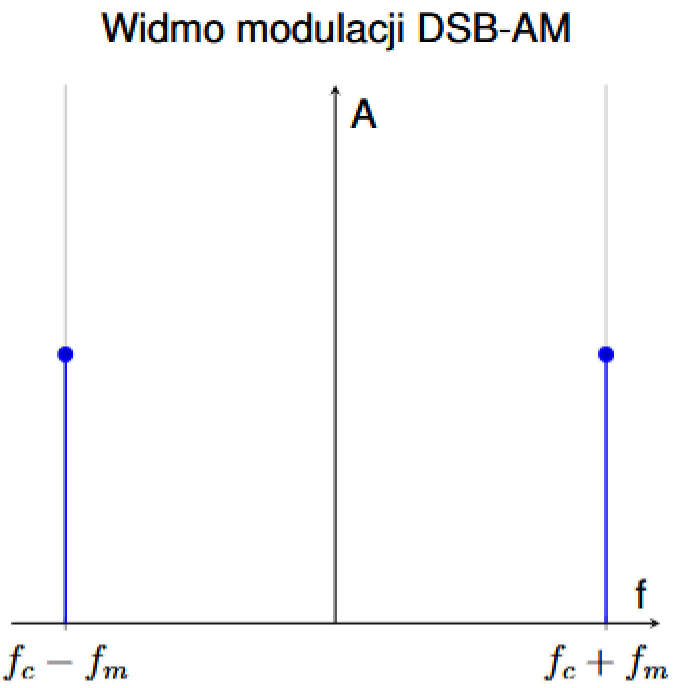
$$s(t) = m(t) \$$cos(2\pi f_c t)$$. where: * $$s(t)$ is the output signal, which is the result of modulation,

- $m(t)$ is the information (modulating) signal,
- f_c is the carrier frequency,
- * $cos(2\pi f_c t)$ is the carrier function at frequency f_c.

In this modulation:

- The carrier signal $\cos(2\pi f_c t)$ is multiplied by the signal $m(t)$, which represents the information to be transmitted.
- Since no carrier wave is transmitted in this version of DSB-AM (so there is no fixed component of the carrier in the signal), the signal is purely a combination of sidebands resulting from the modulation.

The spectrum of a DSB-AM signal:



SSB-AM modulation

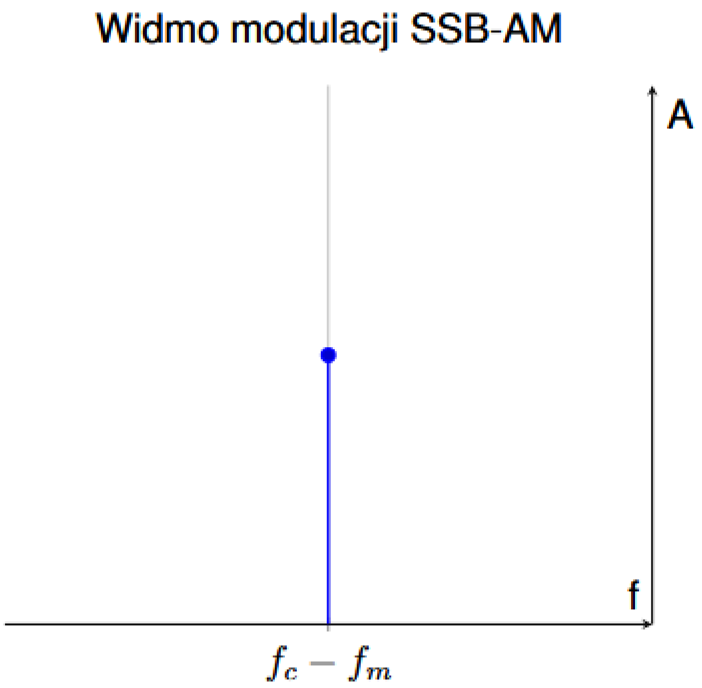
Single sideband modulation (SSB-AM) allows only one sideband to be transmitted, saving bandwidth. The expression for a signal modulated in this form is described by the formula:

$$s(t) = \frac{1}{2} \Ùbig[m(t) + j Ùhat{m}(t) Ùbig] e^{j2{frac{m} f_c t} + c.c.$$

Where:

- $s(t)$ is the output signal,
- $m(t)$ is the information (modulating) signal,
- $hat{m}(t)$ is the Hilbert transform of the signal $m(t)$,
- f_c is the carrier frequency,
- $e^{j2{f_c t}$ is the carrier component in complex form,
- $c.c.$ denotes the complex conjugate component.

In this modulation, instead of two sidebands, only one sideband is transmitted (transmitted in real and complex part form), thus saving bandwidth.



The Hilbert transform is a mathematical operation that, for a signal $m(t)$, generates a signal $hilbertian signal ($hat{m}(t)$), called a Hilbertian signal. It is a transformation that transforms the real signal into a signal of the same amplitude but shifted in phase by $90^\rc$.

Mathematically, the Hilbert transform of ${hat{m}(t)$ is defined as a spline integral:

$$\hat{m}(t) = \frac{1}{} \$$text{P.V.} \{int_{infty}^} \frac{ }{t - \nfty} dint$$. where P.V. denotes the Cauchy principal value and $m(t)$ is the input signal. The Hilbert transform is widely used in signal analysis, including in SSB-AM modulation, where it allows for a signal that, when combined with the original signal $m(t)$, forms a single sideband in the modulation process.
The splicing integral is a mathematical operation that combines two signals into one new signal. It is a fundamental concept in signal and systems analysis, especially in filter theory and signal processing.

For two functions $f(t)$ and $g(t)$, the spline integral $(f * g)(t)$ is defined as:

$$(f * g)(t) = \int_{infty}^{\infty} f(\tau) g(t - \tau) d\tau$$.

In the context of the Hilbert transform, the spline integral is used to calculate the signal ${hat{m}(t)$, which is the Hilbert transform of the signal $m(t)$. Mathematically this is written as:

$$hat{m}(t) = \frac{m}(t)$. \¢int_{infty}^infty \frac{ }{t - \fty} d$hat{m}(t).

In this case, $m(\tau)$ is the input signal and ${frac{ (t - \tau)}$ is the function we use to compute the Hilbert transform. It is worth noting that this expression is a spline form of the function $\frac{1}{\pi t}$, also known as the Hilbert function.

The spline integral in this case 'shifts' the signal $m(t)$ in time, creating a new signal $\hat{m}(t)$ that is phase-shifted by 90° relative to $m(t)$, which is a key feature of the Hilbert transform.

This creates a signal that is used in SSB-AM modulation to transmit one sideband.
The carrier component in complex form is the expression $e^{j2\pi f_c t}$, which represents the carrier in complex form. In this form, the carrier is written as a complex number, where j is the imaginary unit and f_c is the carrier frequency. The component $e^{j2\pi f_c t}$ describes the carrier wave, which has a frequency f_c and is expressed as an exponential function. With the complex form, it is easier to manipulate the phase and amplitude of the signal, which is particularly useful in signal analysis and modulation.
In contrast, *$c.c.$* is an abbreviation for complex conjugate. This means that, in addition to the component $e^{j2| f_c t}$ in the expression for the signal $s(t)$, its complex coupling is added, i.e. $e^{-j2| f_c t}$. The complex coupling consists of a change of sign at the imaginary unit j. In the context of SSB-AM modulation, the composite coupled component ensures that the signal will have a real value, since the sum of the composite components and their coupling gives the real result.

VSB-AM modulation

VSB-AM (Vestigial Side Band) modulation is used in television broadcasting, where part of one side band is attenuated to save bandwidth while still being able to receive the full information. It is a type of amplitude modulation in which only part of one of the side bands (usually the bottom band) is transmitted, while the rest of the band is attenuated by a suitable filter. This reduces the bandwidth of the signal compared to traditional AM modulation, which is advantageous in television transmission where bandwidth efficiency is crucial.

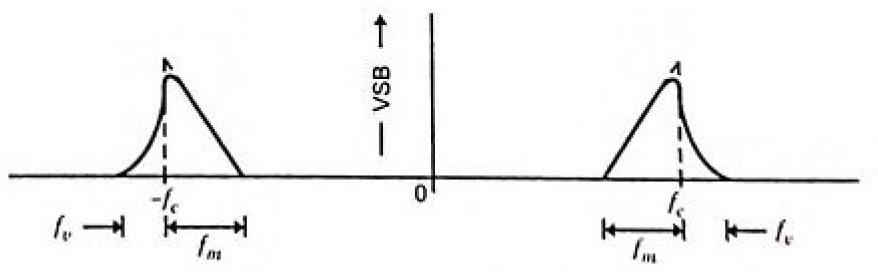
The expression for the signal spectrum $S(f)$ in VSB-AM modulation is written as:

$$S(f) = M(f) H(f),$$.

where:

- $M(f)$ is the spectrum of the information signal, - $H(f)$ is the transfer function of a filter that attenuates part of one sideband, leaving only the 'residual' part of that sideband.

The $H(f)$ filter is responsible for removing the redundant parts of the sideband, thus saving bandwidth, but in a way that does not lead to the loss of relevant information.



Spectrum diagram of a VSB-AM signal

Frequency modulation

Frequency modulation (FM) is a technique in which the frequency of a carrier is modulated by an information signal $m(t)$. Unlike amplitude modulation, where the amplitude of the carrier changes, in frequency modulation its frequency changes depending on the value of the modulating signal. The formula for an FM signal can be written as:

$$s(t) = A \cos \left(2 \pi f_c t + \Delta f \sin(2\pi f_m t) \right)$$.

where:

- A is the amplitude of the carrier, - f_c is the carrier frequency, - $Delta f$ is the maximum frequency deviation (deviation frequency), - f_m is the frequency of the modulating signal $m(t)$.

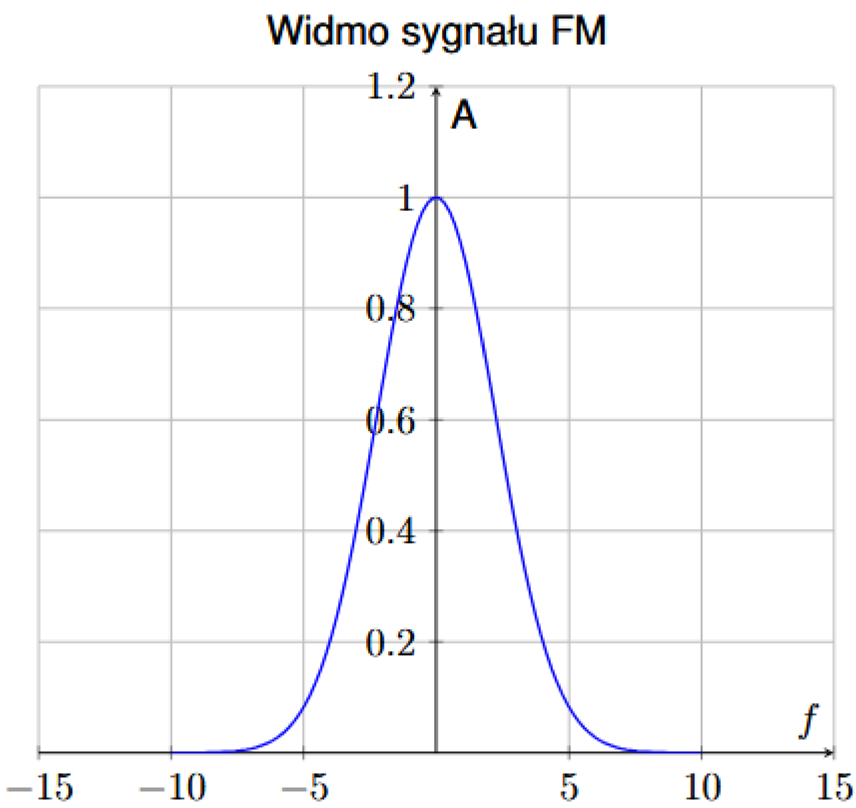
In the spectrum of an FM signal, depending on the value of $Delta f$, many sidebands appear distributed around the carrier frequency f_c. The formula for the FM signal spectrum is more complex, but its general form can be written as:

$$S(f) = \sum_{n=-\infty}^{\infty} J_n(\beta) \cdot \delta(f - f_c - n f_m)$$.

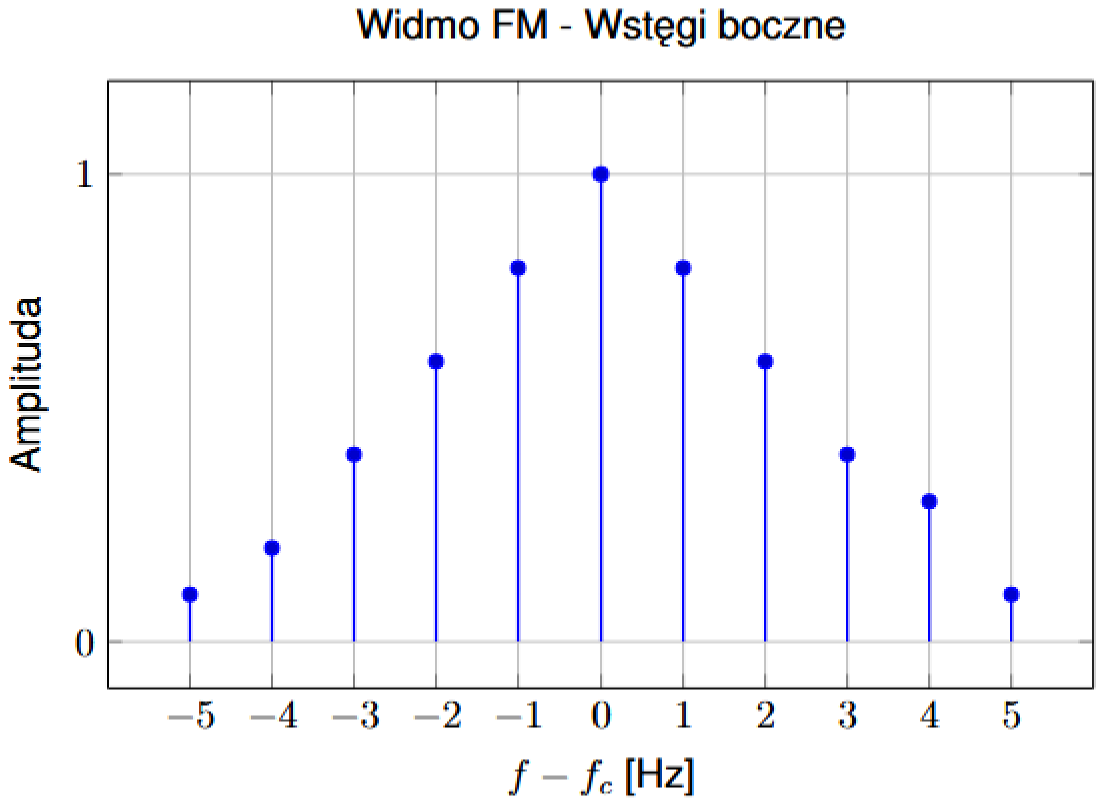
Where:

- $J_n(\beta)$ is a Bessel function of the first kind with index n, and $\beta = \frac{\delta f}{f_m}$ is the modulator index (deviation factor), - $\delta(f)$ is Dirac's delta, representing the sidebands in the spectrum.

For small values of ${beta$ (narrowband modulation), the spectrum consists mainly of the first sideband, while for large ${beta$ (broadband modulation), many sidebands appear.



FM signal spectrum diagram



FM signal spectrum with sidebands

With frequency modulation, the spectrum of an FM signal extends over a large bandwidth, especially for large $$-delta f$ values. This means that the FM signal is more resistant to interference compared to AM and SSB-AM signals, but requires a larger transmission bandwidth.

Phase Modulation

Phase modulation (PM) is a technique in which it is the phase of the carrier that is modulated by the information signal $m(t)$. Unlike amplitude modulation (AM), in which the amplitude of the carrier changes, in phase modulation its phase changes in response to the modulating signal. The output signal in phase modulation is described by the formula:

$$s(t) = A \cos \left(2 \pi f_c t + \phi(t) \right)$$.

where:

- A is the amplitude of the carrier, - f_c is the carrier frequency, - $\phi(t)$ is the phase function that depends on the modulating signal $m(t)$.

Typically, the phase is related to the modulating signal by the formula:

$$\phi(t) = k_p \cdot m(t)$$.

where k_p is the phase amplification factor, which determines how much the modulating signal changes the phase of the carrier.

The spectrum of a PM signal is similar to that of frequency modulation, except that the amplitudes of the sidebands depend on the first Bessel functions, similar to frequency modulation (FM). Changing the phase of the signal causes shifts in the spectrum, which are represented by different sidebands with amplitudes of $J_n(\beta)$, where β is the phase deviation factor (which determines how strongly the phase of the carrier changes depending on the modulating signal).

Thus, the spectrum of the PM signal is expressed as:

$$S(f) = \sum_{n=-infty}^ J_n(\beta) \cdot \delta(f - f_c - n f_m)$$.

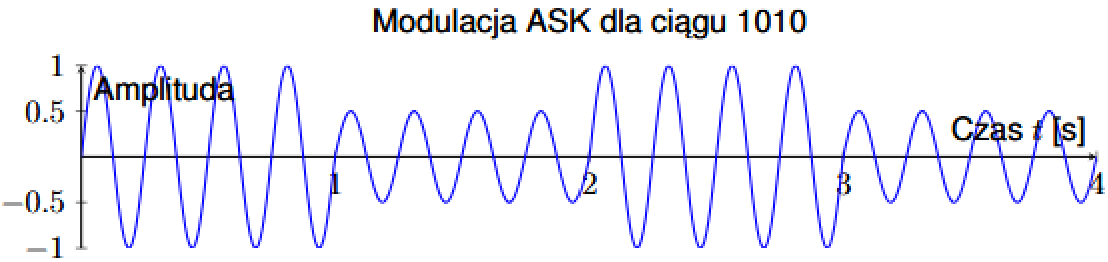
where $J_n(\beta)$ are Bessel functions of the first kind with index n, and $\beta = k_p \cdot m_{\text{max}}$$, where m_{max} is the maximum value of the amplitude of the modulating signal $m(t)$.

As with frequency modulation, in phase modulation for small values of ${beta$ only the first sidebands appear, while for larger values of ${beta$ the spectrum becomes wider, spreading over many sidebands.

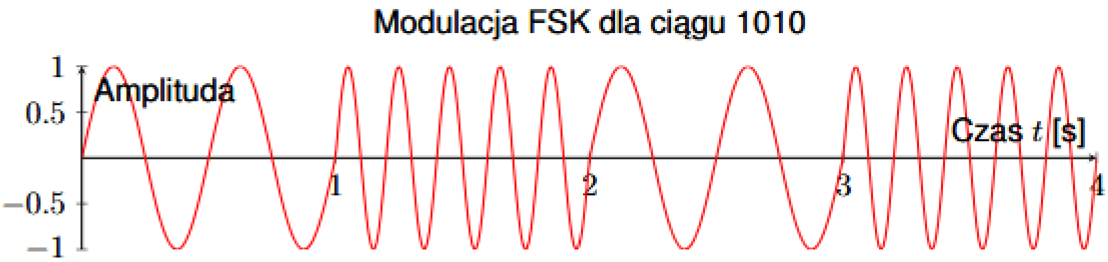
Graphs the same as for FM modulation

Digital modulations

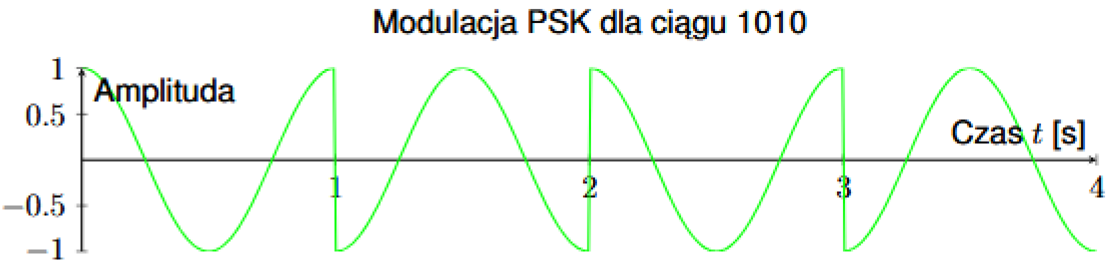
Digital modulations are used in the transmission of digital signals, where information is transmitted by changing the parameters of the carrier, such as amplitude, frequency or phase. With these techniques it is possible to transmit data efficiently in telecommunications systems.



((a)) ASK



((b)) FSK



((c)) PSK

Digital modulations ASK, FSK, PSK for coded string 1010

ASK modulation (Amplitude Shift Keying)

Amplitude shift keying (ASK) modulation is one of the basic digital modulations in which the amplitude of the carrier is modulated according to the value of the digital bits. In ASK modulation, we have two possible amplitudes:

$$s(t) = \Ùbegin{cases} A_1 \cos(2 \pi f_c t) & \text{for bit 1} \\ A_2 \cos(2 \pi f_c t) & \text{for bit 0}. \■end{cases}$$.

where A_1 and A_2 are the different amplitudes corresponding to the logic 1 and 0 states, and f_c is the carrier frequency.

With this modulation, the signal has only one sideband, whose amplitude is changed according to the value of the transmitted bit.

FSK modulation (Frequency Shift Keying)

Frequency shift keying (FSK) modulation involves changing the carrier frequency according to the value of the bit. In the case of two-phase FSK modulation, we have two frequencies f_1 and f_2, which correspond to bit values 0 and 1. The formula for the signal in this modulation is:

$$s(t) = \■begin{cases} A \cos(2 \pi f_1 t) & \text{for bit 1}. \\ A \cos(2 \pi f_2 t) & \text{for bit 0}. \■end{cases}$$.

where f_1 and f_2 are different carrier frequencies and A is the amplitude of the carrier.

FSK modulation is more resistant to interference and noise compared to ASK because the change in frequency is less sensitive to changes in signal amplitude.

PSK modulation (Phase Shift Keying)

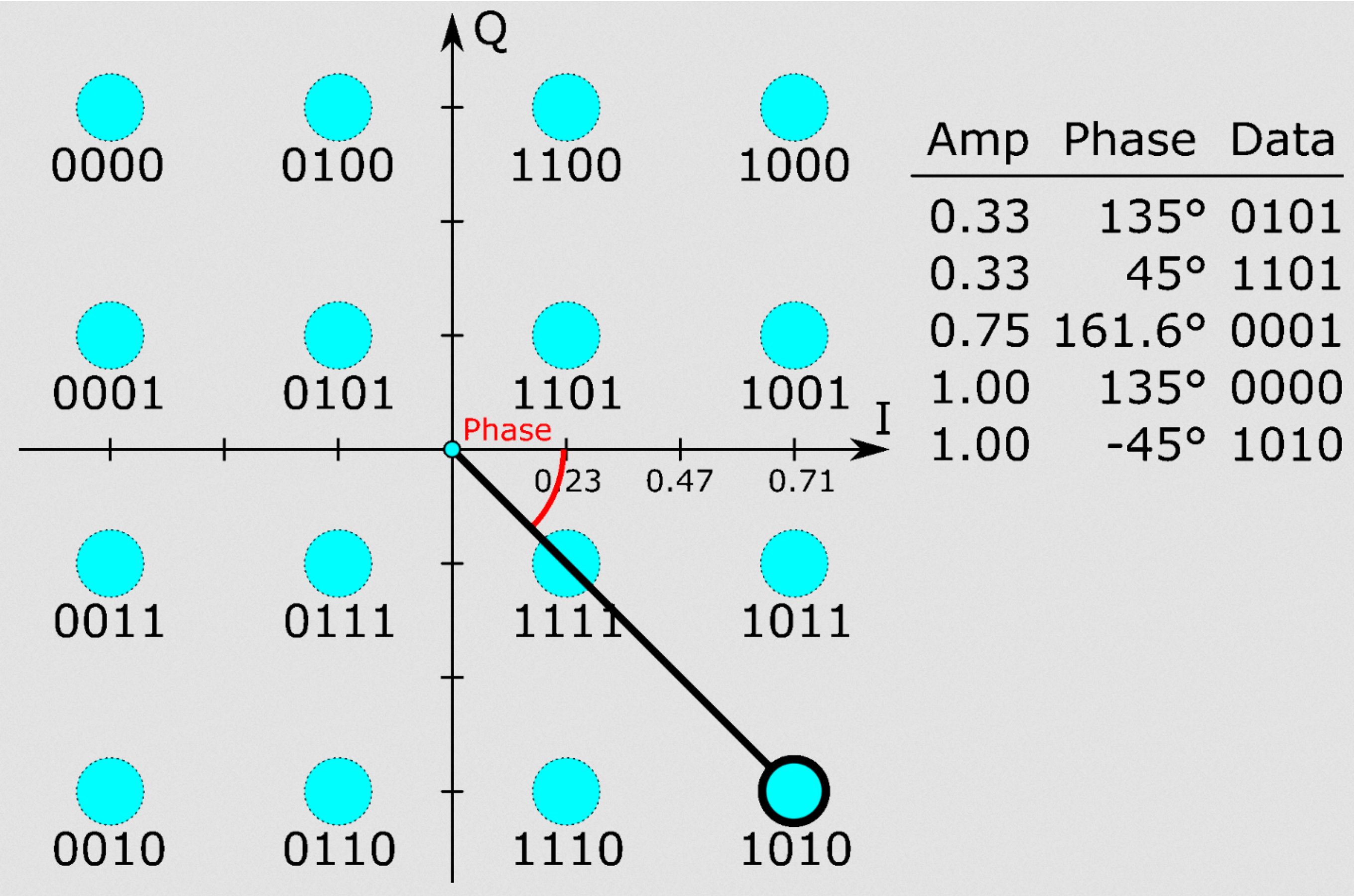
Phase shift modulation (PSK) involves changing the phase of the carrier depending on the transmitted bits. In the simplest version of PSK (BPSK, Binary PSK), we have two possible phases, e.g. 0 and $, corresponding to bits 0 and 1. The formula for a PSK signal is:

$$s(t) = A \cos \left(2 \pi f_c t + \phi_k \right)$$.

where $\phi_k \in \n 0, \pi \nright)$$ is the phase of the carrier, which varies depending on the value of the bit (0 or 1).

In the case of PSK, an extended version is also possible, such as QPSK, where we modulate four different phases (e.g. $0, \frac{\pi}{2}, \pi, \frac{\pi}{2}$).

Quadrature modulations



Amplitude-phase diagram of a 16QAM signal

Quadrature modulation is a technique in which information is transmitted by modulating both the amplitude and phase of the carrier. The most common examples are QAM (Quadrature Amplitude Modulation), where both amplitude and phase are varied in a balanced manner.

With QAM, we can have, for example, QPSK (Quadrature Phase Shift Keying), where two separate phase components are transmitted on a single carrier. The QPSK output signal can be described by the equation:

$$s(t) = A_1 \cos(2 \pi f_c t + \phi_1) + A_2 \cos(2\pi f_c t + \phi_2)$$.

where A_1 and A_2 are the amplitudes and ϕ_1 and ϕ_2 are the phases, which can take different values depending on the combination of bits (e.g. for QPSK four different phase combinations).

Quadrature modulations, such as QAM and QPSK, offer greater efficiency in bandwidth utilisation because more information can be transmitted per carrier.

Pulse modulations

Pulse modulations are techniques in which information is transmitted using pulses with specific properties such as amplitude, duration or position in time. They are used in a variety of communication applications, including telecommunications, radar systems and digital data transmission systems. The purpose of these modulations is to achieve efficient transmission of information within a limited bandwidth and to provide immunity to interference. The main types of pulse modulation include: PAM, PWM, PPM and PCM.

PAM modulation (Pulse Amplitude Modulation)

Description

Pulse amplitude modulation (PAM) is one of the simplest forms of modulation, in which the amplitude of the pulses is varied according to the information to be transmitted. In this method, the amplitude of successive pulses is proportional to the value of the information signal.

Mathematical formula

Mathematically, the signal in PAM modulation can be written as: $$s(t) = \sum_{n=0}^{N-1} m_n \cdot p(t - nT),$$ where: - m_n - amplitude of the pulse on the nth sample, - T - sampling period, - $p(t)$ - pulse function (e.g. rectangular function).

Operating principle

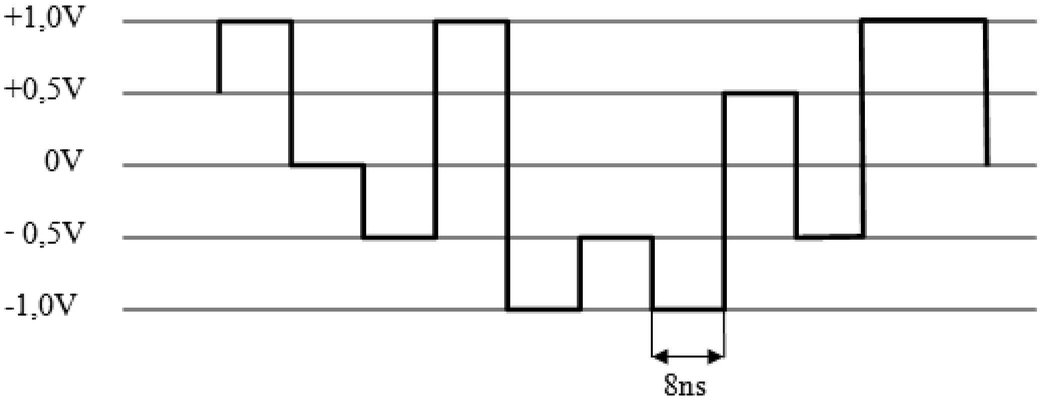
In PAM modulation, the pulse amplitude is directly dependent on the value of the information signal. The change in pulse amplitude can be realised discretely (e.g. for binary signals) or continuously (for analogue signals).

Application

PAM modulation is mainly used in the transmission of digital and analogue signals in systems where the bandwidth is not strictly limited, such as telecommunications systems.

Diagram

A PAM signal in the time domain is a series of pulses of varying amplitude. A graph of an example PAM signal is shown below:



Graph of PAM modulation over time

Pulse Width Modulation (PWM)

Description

Pulse width modulation (PWM) involves varying the pulse duration depending on the value of the information signal. The pulse frequency is constant and only the pulse width changes.

Mathematical formula

In the case of PWM, the signal is described by the formula: $$s(t) = \sum_{n=0}^{N-1} m_n \cdot u(t - nT),$$ where: - m_n - pulse amplitude, dependent on pulse width, - $u(t)$ - time-varying rectangular function.

Operating principle

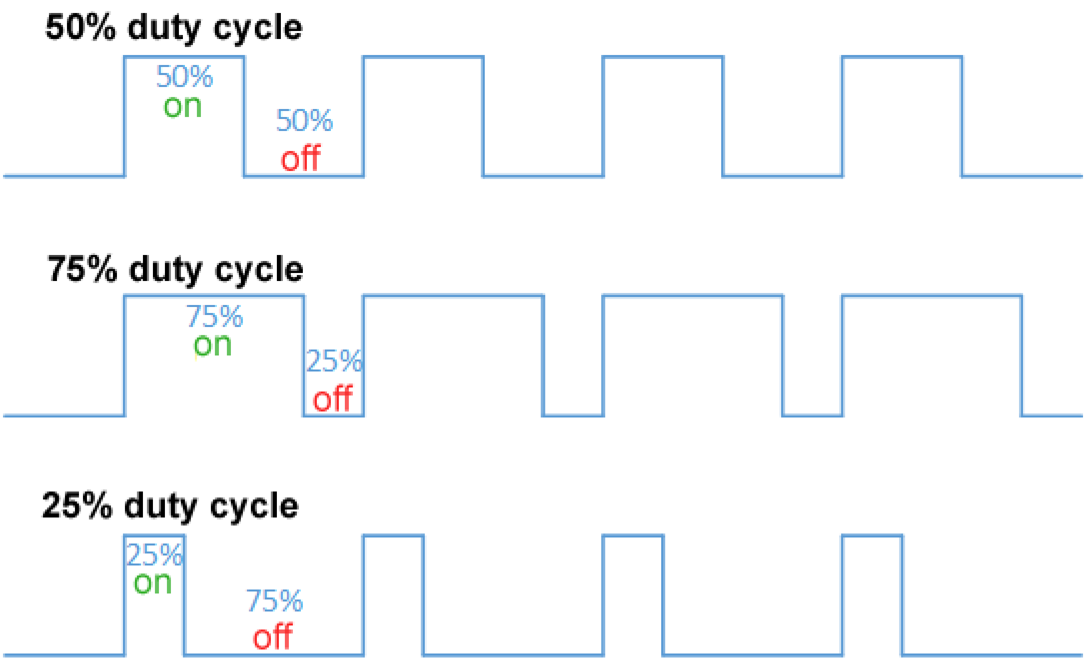
In PWM modulation, the pulse width is a function of the value of the information signal. By varying the pulse width, different signal levels can be encoded, allowing more precise transmission.

Application

PWM is commonly used in motor control systems and also in electronics for power regulation (e.g. in power supplies). In addition, PWM modulation is used in audio and video transmission.

Graph

A PWM signal in the time domain is a series of rectangular pulses of varying width:



Graph of PWM modulation with varying pulse lengths

Pulse Position Modulation (PPM)

Description

Pulse position modulation (PPM) consists of shifting the time of occurrence of a pulse depending on the value of the information signal. In this method, the amplitude of the pulse remains constant, but its position in time is changed.

Mathematical formula

The signal in PPM modulation can be written as: $$s(t) = \sum_{n=0}^{N-1} m_n \cdot p(t - nT - \Delta t_n),$$ where: - m_n - pulse amplitude, - Δt_n - pulse delay depending on the information signal.

Operating principle

In the PPM, by changing the time of pulse occurrence (position), we encode information. The time between pulses is constant, while their positions are a function of the signal.

Application

PPM modulation is used in systems that need to transmit data in an interference-resistant manner, such as optical communication systems.

Graph

A time-domain PPM signal is a series of pulses whose position in time varies according to the information.

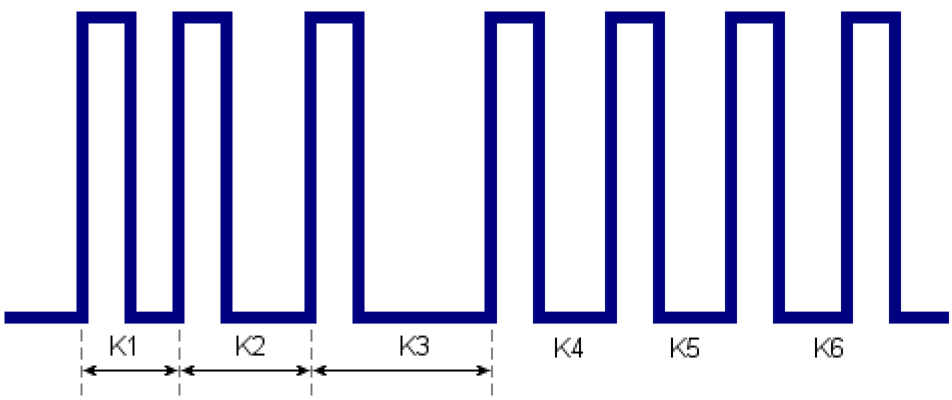


Diagram of PPM modulation in time

PCM modulation (Pulse Code Modulation)

Description

Pulse code modulation (PCM) is a digital modulation technique that samples an analogue signal and converts it into a discrete sequence of pulses. Each pulse represents a specific sample value of the analogue signal.

Mathematical formula

We write the PCM signal as: $$s(t) = \sum_{n=0}^{N-1} m_n \delta(t - nT),$$ where: - m_n - signal sample value, - $delta(t)$ - Dirac function (unit impulse).

Operating principle

In PCM, the analogue signal is sampled at specified intervals and then each sample is encoded in digital form (usually in binary form). Digital pulses are transmitted, which represent the signal levels at a given time.

Application

PCM is widely used in digital communication systems, including digital telephony, audio compression (e.g. MP3) and sound systems.

Graph

A PCM signal is a sequence of pulses that represent digital samples of an analogue signal:

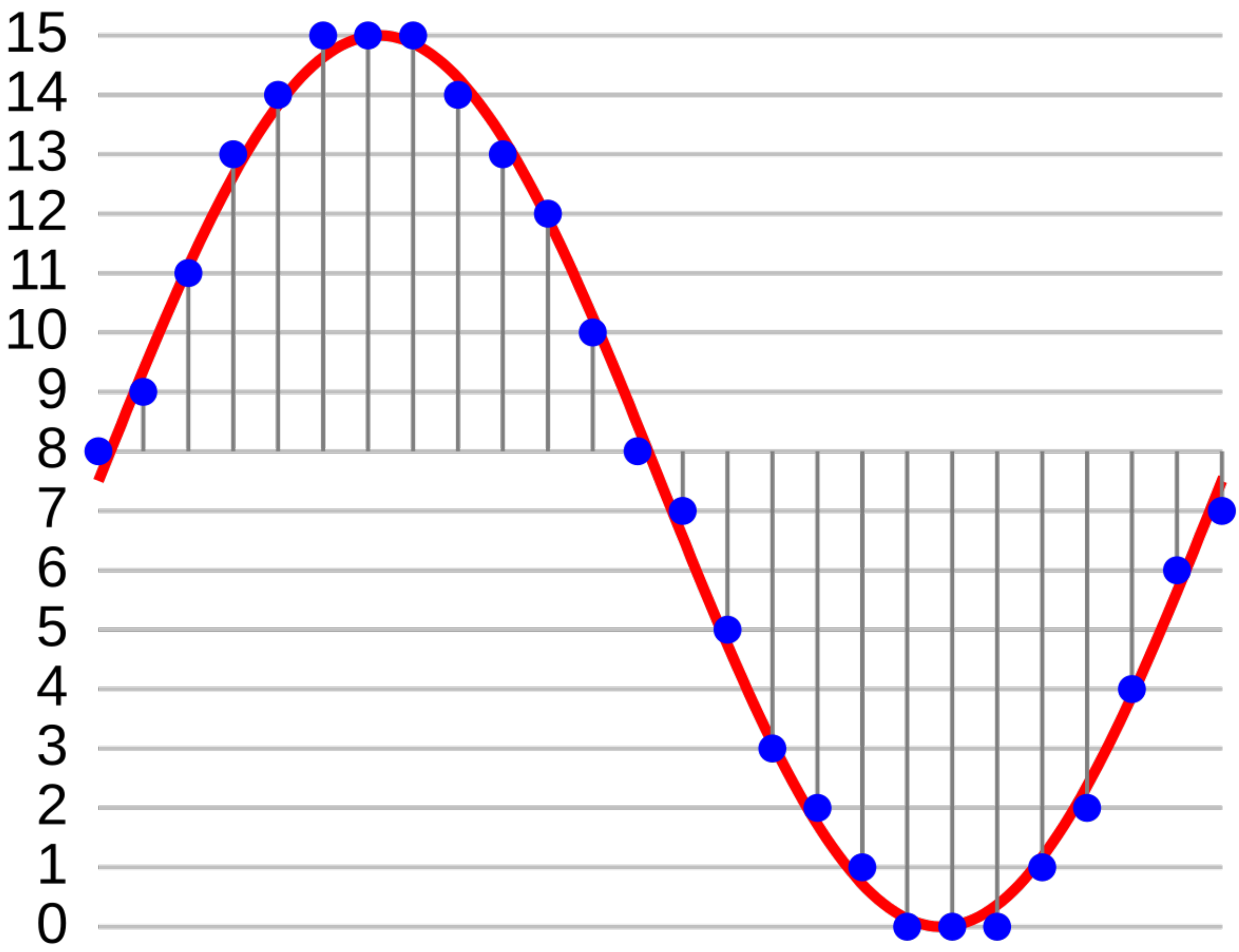


Diagram of PCM modulation over time (signal after quantisation process)

Transimpedance: Application, Operation and Physical Phenomena

Transimpedance is an important concept in electronics, particularly in the context of transimpedance amplifiers (TIA - Transimpedance Amplifier). It is a parameter that describes the conversion of input current to output voltage. Formally, the transimpedance Z_T is defined as:

$$Z_T = \frac{V_{out}}{I_{in}}$$.

which means that the unit of transimpedance is the ohm (Ω).

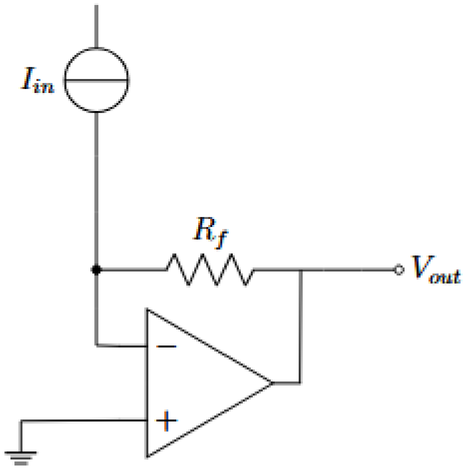
Application of

Transimpedance amplifiers are widely used in systems that process current sensor signals, such as:

- Photodiodes in optoelectronic systems (e.g. fibre optic receivers, optical detectors).
- Current probes in oscilloscopes.

- Biometric systems and chemical sensors.

Operation and Mathematical Modelling



The most commonly used transimpedance amplifier circuit is an operational amplifier with a feedback resistor R_f to provide the appropriate signal conversion. The circuit diagram is as follows:

(0.5,0) node[op amp] (opamp) (opamp.+) - (-2,-0.5) node[ground] (opamp.-) - (-1,0.5) to[short, -*] (-1,1.5) to[R, l=R_f] (2,1.5) to[short, -o] (3,1.5) node[right]V_{out} (-1,1.5) to[short] (-1,3) to[I, l=I_{in}] (-1,5) node[above] (opamp.out) - (1.70,1.5);

The node analysis at the input of the operational amplifier shows that the voltage at the inverting input is $0V$ (ideal operational amplifier model). Applying Ohm's law to the feedback resistor:

$$V_{out} = -I_{in} R_f$$

which shows that the transimpedance is:

$$Z_T = -R_f$$

Meaning of the minus sign at R_f

The minus sign in the expression $Z_T = -R_f$$ means that the transimpedance amplifier performs a phase inversion of the signal. This means that if the input current I_{in} is positive, the output voltage V_{out} will be negative and vice versa. This is a consequence of an operational amplifier operating in an inverting configuration, where the voltage at the inverting input is zero (virtual ground) and the feedback through R_f causes the polarity of the signal to be reversed.

Physical Phenomena and Limitations

In practical applications, several important effects must be taken into account:

- **Johnson-Nyquist noise** - the feedback resistor generates thermal noise, which can limit the sensitivity of the circuit.
- **Input capacitance** - photodiodes and other current sensors have their own capacitance C_d, which affects the bandwidth of the circuit.
- **Frequency response** - in reality, an operational amplifier has a finite voltage rise rate (slew rate) and limited bandwidth, which affects the system's response to time-varying signals.

To improve the stability and frequency response of a transimpedance amplifier, pole compensation is often used by adding a capacitor in parallel with resistor R_f, which reduces oscillations and increases the stability of the circuit.

Summary

Transimpedance amplifiers are a key component of many current sensor signal processing circuits. Their design requires consideration of both electronic parameters and physical effects such as noise and frequency limitations. Correct mathematical analysis allows circuits to be optimised and ensure stable operation in real-world applications.

BalUn circuits (BalUn's own symmetisers)

A BalUn (Balanced to Unbalanced) is an impedance transformer used to convert signals between balanced and unbalanced circuits. Its main function is to provide an appropriate impedance transformation between the two types of circuits.

History of the name

The name BalUn is derived from the English words *Balanced* (balanced) and *Unbalanced* (unbalanced). These circuits are widely used in radio communications and signal transmission, where balanced lines (e.g. coaxial cable) are used to transmit signals in antenna systems, and unbalanced circuits (e.g. circuits with an impedance of 50 $mega$) are commonly used in radio equipment.

The importance of transimpedance in BalUn

The transimpedance in a BalUn circuit is a key parameter, as it determines the circuit's ability to convert a signal from one form to another while still maintaining the correct impedance. In practice, the transimpedance of a BalUn depends on the type of transformer used and its characteristics. Modern BalUn's used in RF (Radio Frequency) equipment must be designed to minimise signal loss, ensuring low transimpedance values with appropriate parameters.

Calculation of BalUn parameters

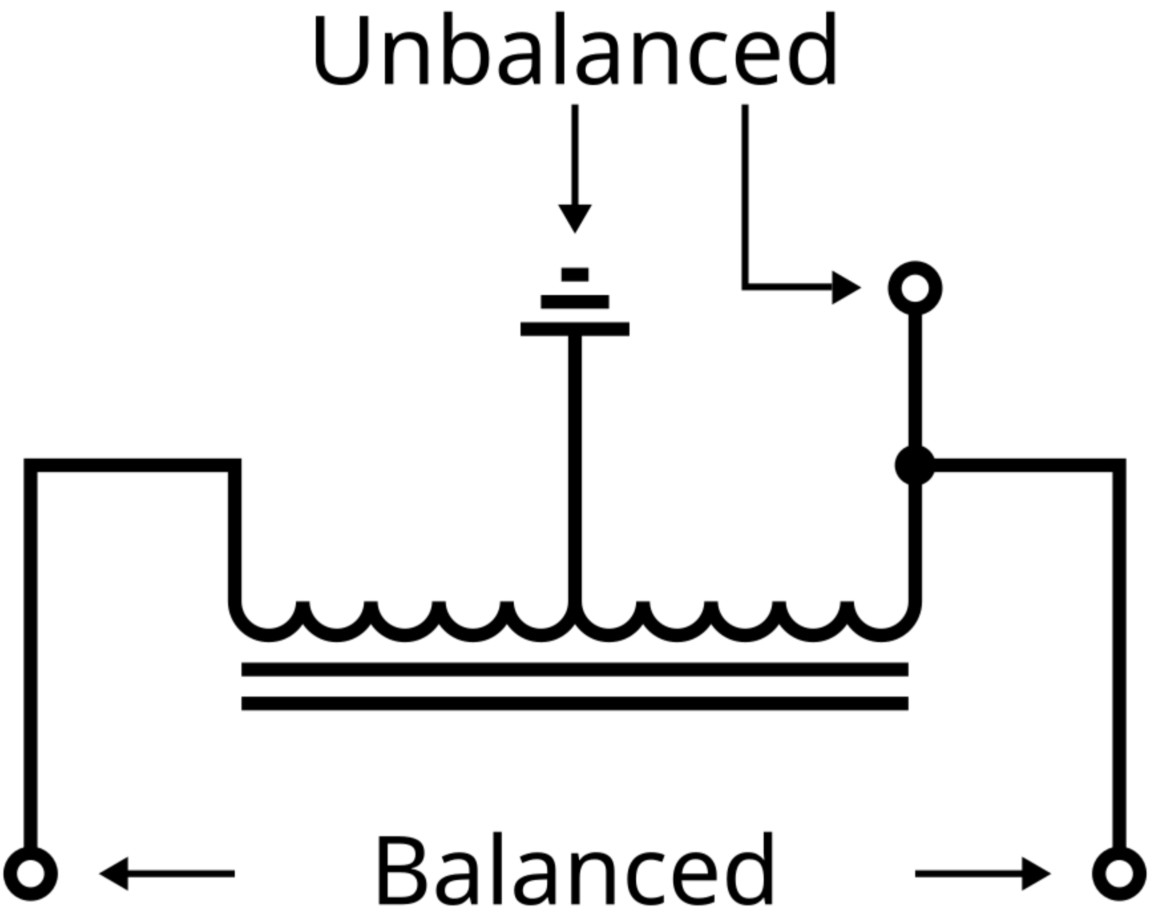
BalUn parameters depend on several factors, such as the impedance ratio (e.g. 50 $Omega$ to 200 $Omega$), the signal frequency and the shape of the transformer. To calculate the transimpedance, formulas depending on the transformer type are used:

$$Z_{text{balun}} = \$$sqrt{Z_text_in} \Z_text_out}$$. where: - Z_{in} is the input impedance, - Z_{out} is the output impedance.

For the BalUn circuit, the impedance transformation takes place in a proportional manner, with the transformer playing the role of lowering or raising the impedance depending on the circuit design.

Schematic of the BalUn circuit

A simple schematic of the BalUn circuit is shown below:



BalUn circuit diagram

UnUn circuit

The UnUn (Unbalanced to Unbalanced) circuit is also an impedance transform circuit, but in this case both ends of the circuit are unbalanced. These circuits are mainly used for impedance matching in circuits where both components of the system are built on an unbalanced line.

History of the name

As with the BalUn circuit, the name UnUn is derived from the English words *Unbalanced* and

Unbalanced. In this circuit, the main purpose is to match the impedance between two unbalanced circuits, e.g. between two devices that operate at different impedance levels (e.g. $75 $Omega$ and $50 $Omega$). This circuit is commonly used in telecommunications networks and audio systems.

The importance of transimpedance in UnUn

The transimpedance in the UnUn circuit plays a similar role to that in the BalUn circuit, as it allows for efficient signal conversion between different unbalanced impedances. This is important in the context of impedance matching for correct signal transmission and the avoidance of power losses.

Calculation of UnUn parameters

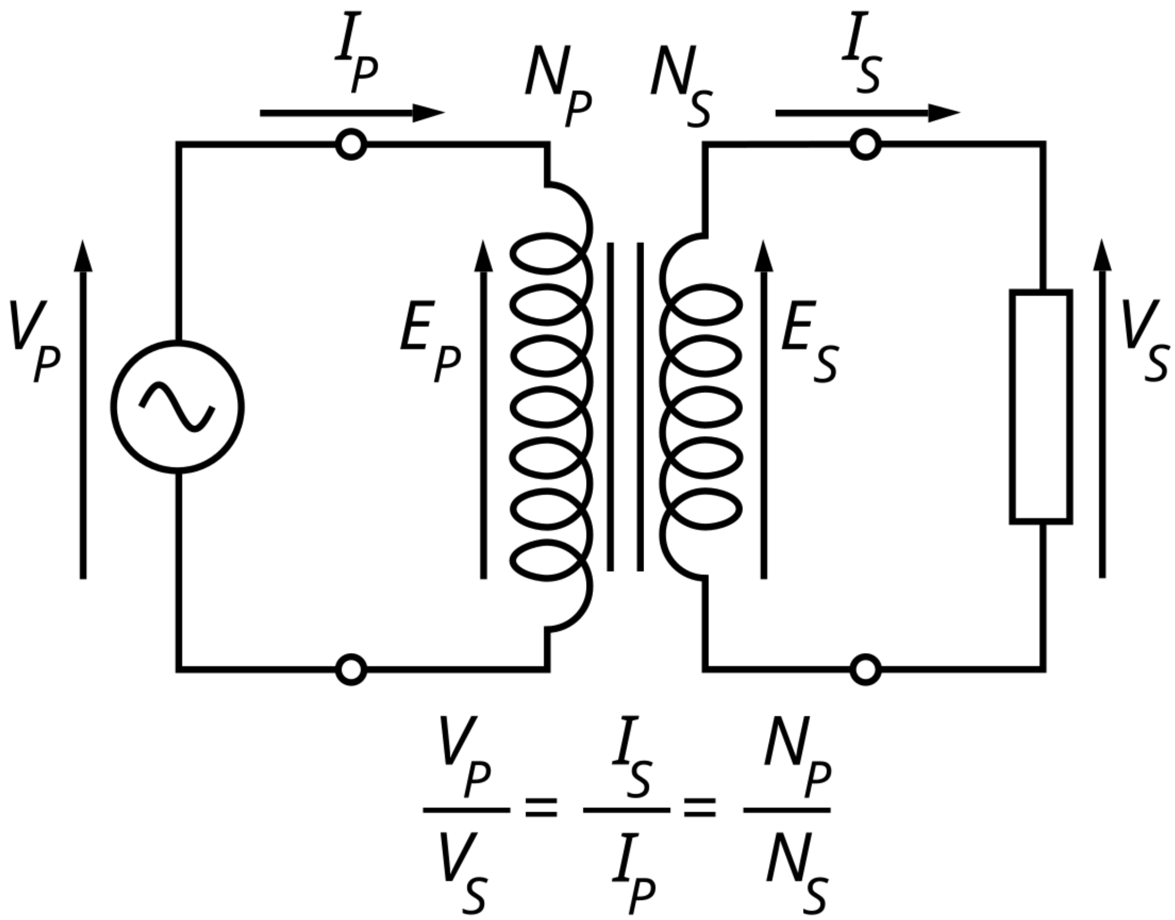
As with BalUn, the calculation of the UnUn circuit parameters depends on the input and output impedances. The transimpedance formula for the UnUn circuit is as follows:

$$Z_{{ununn}} = \$$frac{Z_text_in} \cdot Z_{\text{out}}}{Z_{\text{in}} + Z_text_out}$$. where: - Z_{in} is the input impedance, - Z_{out} is the output impedance.

UnUn circuits can be used to connect devices of different impedances, making them particularly useful in audio and telecommunications systems.

UnUn circuit diagram

An example of a UnUn circuit diagram is shown below:



UnUn circuit diagram

Smith diagrams and their application in impedance matching

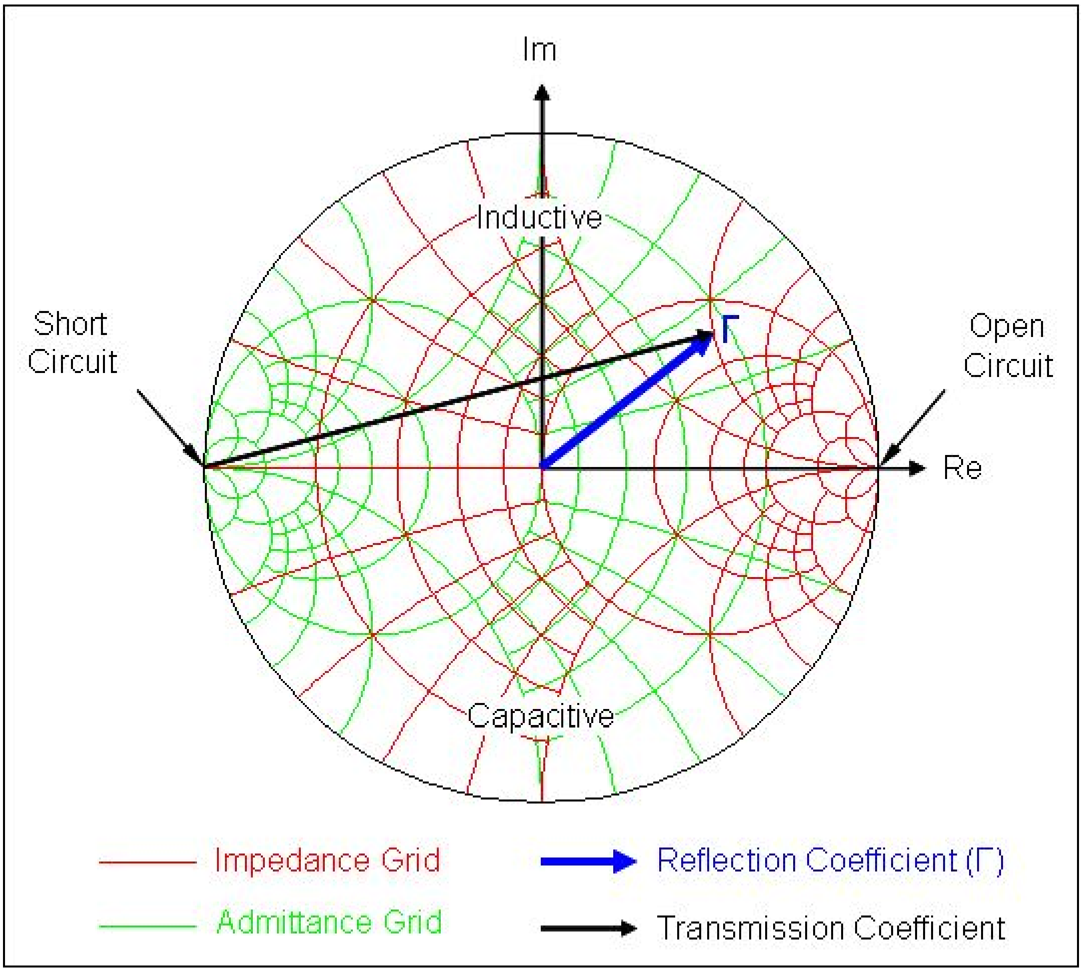


Illustration to aid understanding of how a smith chart works

The Smith chart is a graphical tool used in telecommunications and electronics for impedance analysis and impedance matching in RF, microwave and other RF circuits. It is a special case of the composite chart, where both impedances and admittances are represented. Due to its design, the Smith chart allows for an easy representation of the impedance properties of circuits, which facilitates the design of RF circuits.

Basic concepts

The impedance Z of an electrical circuit is a complex quantity and has the form: $$Z = R + jX$$ where:

- R - resistance,
- X - reactance,
- j - imaginary unit.

Smith's diagram represents impedances in a complex system, where the axes correspond to the resistive and reactance components. The main elements of the diagram are the circles corresponding to the resistance values, and the circles corresponding to the reactance values.

Application of the Smith diagram to impedance matching

Impedance matching is the process of adjusting the impedance of the source and load to minimise energy loss and ensure maximum power transfer. The Smith chart is a tool to make this matching quick and intuitive. Using a Smith chart for impedance matching involves the following steps:

1. **Determining the impedance of the source and load:** On the Smith chart, we represent the impedances of the source and load as points on corresponding circles, which represent the resistive and reactance values.
2. **Calculation of the fit factor:** We then determine the fit factor (e.g. the reflection coefficient) based on the distance between the points representing the source and load impedances and the centre of the graph. This value indicates the quality of the impedance match.
3. **Selection of the matching element:** The Smith chart allows the selection of suitable matching elements, such as chokes, capacitors or other passive elements, to shift the impedance point towards the centre of the chart. This shift reduces the reflection coefficient and improves the matching efficiency.

An important feature of the Smith chart is that it also allows visualisation of impedance changes as a function of transmission line length. For example, the changing impedance along a transmission line can be represented as a curve on the graph, making it easier to assess the effects of changing line length on impedance matching.

The mathematics behind the Smith diagram

The Smith chart is a tool that is based on complex mathematics and impedance transformations. Its purpose is to represent impedance and admittance graphically, allowing an intuitive understanding of their properties and easy impedance matching. In this sub-section, we will discuss the mathematical basis of the Smith diagram, including how impedance is represented and the relationships between impedance, admittance and reflection coefficient.

Impedance and its representation on the complex plane

Impedance Z is a complex quantity that can be written in the form: $$Z = R + jX$$ where:

- R - the real part, corresponding to the resistance,
- X - imaginary part, corresponding to reactance,
- j - imaginary unit, $j^2 = -1$.

Smith's diagram is based on the representation of this impedance on a unit circle in the complex plane. Impedances normalised to the characteristic impedance of the transmission line Z_0 are represented as a point on this circle, where: $$z = \frac{Z}{Z_0}$$ The representation of the impedance on the Smith diagram takes into account both resistance and reactance, and the circles on the diagram correspond to different values of these parameters.

Admitance

Admitance is the inverse of impedance in AC circuits. It is a complex quantity that describes the ease of current flow through circuit elements. Admitance Y is defined as the inverse of impedance Z: $$Y = \frac{1}{Z} = \frac{1}{R + jX}$$ where:

- R - resistance,
- X - reactance,
- j - imaginary unit.

Admitance is also a complex quantity, the real part of which is conductance (G - the inverse of resistance) and the imaginary part is susceptance (B): $$Y = G + jB$$ In electrical circuits, conductance (G) measures the ability to conduct current, and susceptance (B) is responsible for the response of circuit elements, such as capacitors and chokes, to a varying electric field. Like impedance, admittance is used in transmission line analysis and impedance matching.

Impedance to admittance transformation

Not only impedances but also admittances, which are the inverse of impedances, are represented on the Smith diagram. The admittance Y is calculated as: $$Y = \frac{1}{Z} = \frac{1}{R + jX}$$ The advantage of the Smith chart is that it allows both impedances and admittances to be represented simultaneously, which is particularly useful in transmission line and impedance matching analyses.

The admittance values can be expressed in the form: $$Y = G + jB$$ where:

- G - conductance, the real part of the admittance,
- B - susceptance, the imaginary part of the admittance.

In Smith's diagram for admittance, the axes correspond to similar values, but they represent conductance and susceptance. The circles that represent the admittances are symmetrical about the real axis.

Reflectance

Another important mathematical element associated with the Smith diagram is the reflection coefficient $Gamma$, which describes the degree to which the signal is reflected from the load. The reflection coefficient is defined as the ratio of the amplitude of the reflected wave to the amplitude of the incoming wave and can be calculated from the formula: $$Gamma = \frac{Z_L - Z_0}{Z_L + Z_0}$$ where:

- Z_L - load impedance,
- Z_0 - characteristic impedance of the transmission line.

In a Smith chart, the reflection coefficient is represented as the distance of the point representing the load impedance from the centre of the chart. = = A value of Z_0 indicates no reflection (matched impedance), while Z_01 indicates full reflection (source and load impedances are completely mismatched).

Impedance mapping on a Smith chart

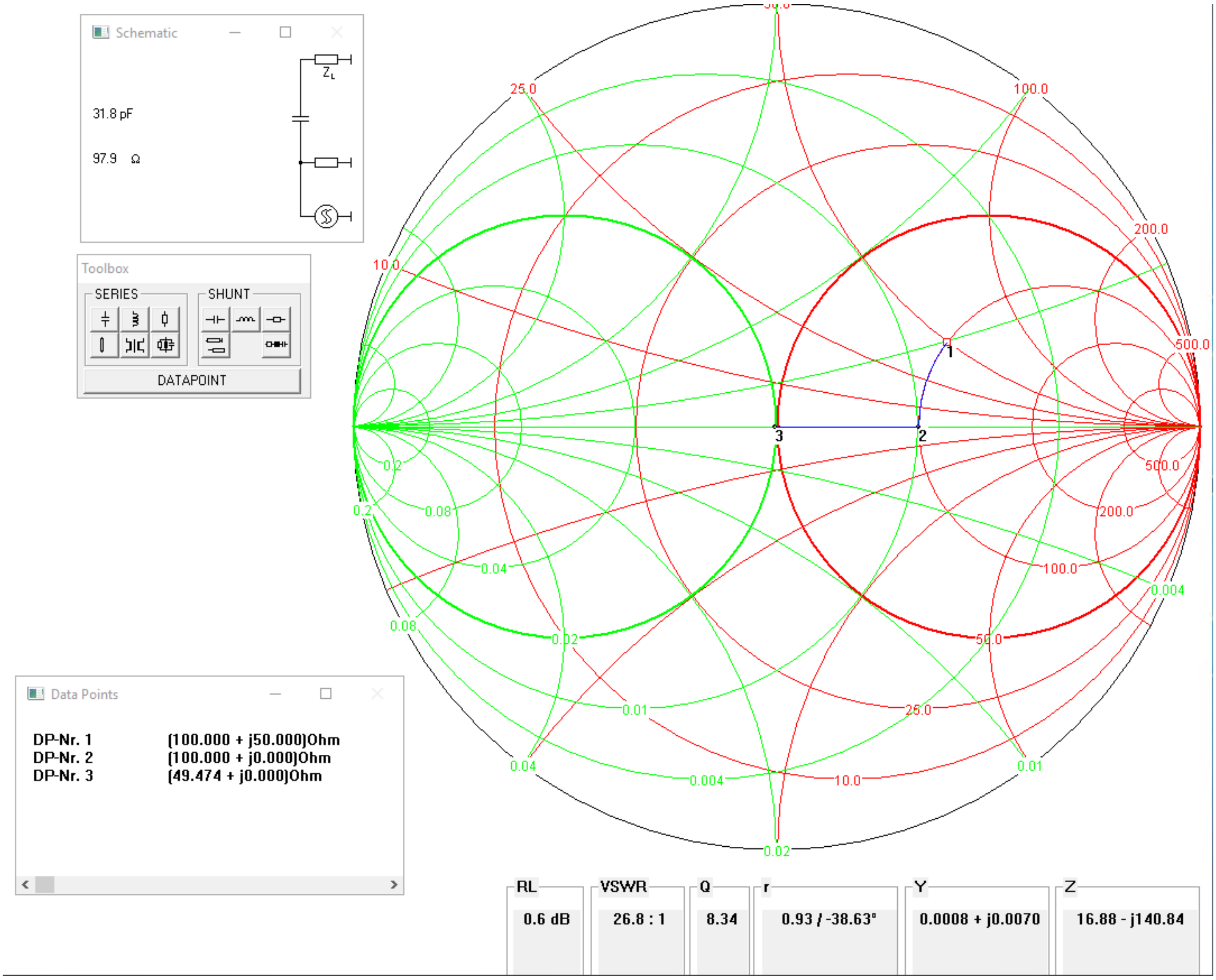
The basic idea of a Smith chart is to map the normalised impedance $z = \frac{Z}{Z_0}$ onto a circular grid. These circles can be interpreted in terms of the distribution of R (resistance) and X (reactance) values. Depending on the value of z, the impedances can be represented on the graph as:

- Points on the circles corresponding to the solid resistance (R-values),
- Points on the circles corresponding to the constant reactance (X values),
- The lines connecting the points on the graph represent changes in impedance due to changes in the length of the transmission line.

All of these transformations allow impedances and admittances to be visualised and manipulated in such a way that impedance matching and maximum power transfer problems can be easily performed.

Example

Suppose we have a circuit with a source impedance $Z_s = 100 + j50 \nmega$. To match the impedance, we can mark a point on the Smith diagram and find a suitable matching element, such as a capacitor or choke, which will shift the impedance towards the centre of the diagram.
The programme used for the presentation below can be downloaded from the link:
http:%%//%%filevista.ardugeek.ovh/public/yq/chart-smitha.exe The link is password protected:
Radio23022025
The programme is unfortunately in demo version.



Application of the smith chart

After the program has performed the calculations, we can see that in order to equalize the impedance to a value of 50 we need to apply a resistor in parallel to the signal source of about 100 and a capacitor in series to the source of about 30pF$

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Vestigial Sideband Modulation System (VSB).. https://www.eeeguide.com/vestigial-sideband-modulation-system-vsb/.

Wikipedia free online encyclopedia. https://www.wikipedia.org/.

Microwaves101.com The world's microwave information source since 2001. https://www.microwaves101.com/encyclopedias/smith-chart-basics.